Contact Mechanics

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Outline

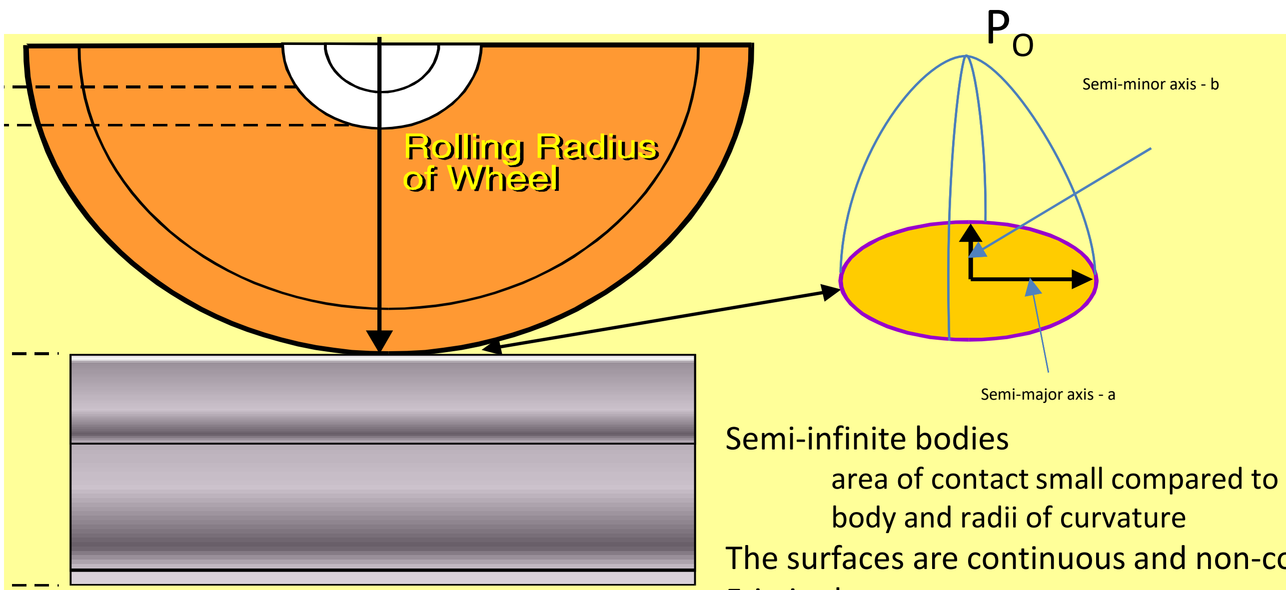
- Contact models - Hertzian contacts
- Pummelling
- Surface Roughness
- Creepage/slip, Creep forces - including thirdbody layers
- Shakedown
- Conformality
- Equivalent Conicity
- Conclusions

Fundamentals of Contact Mechanics

Contact Mechanics

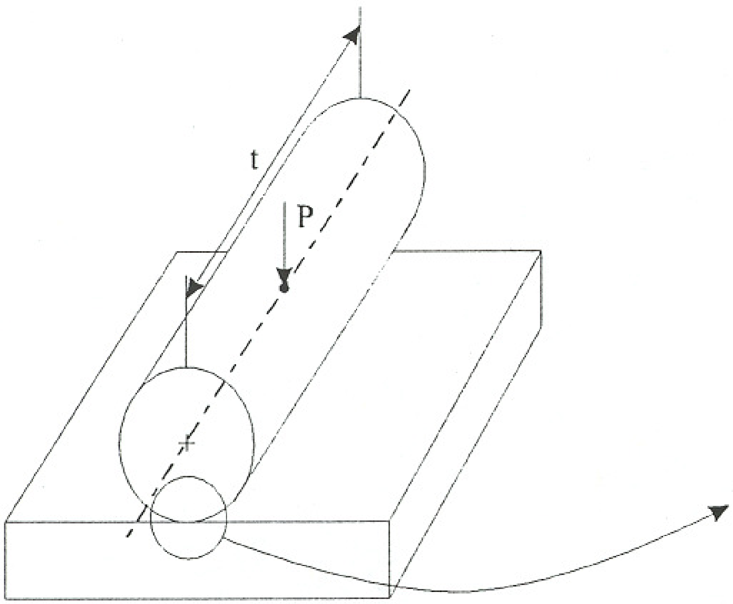
- **Geometry**
  - Point Contact (circle)
  - Line contact (band)
  - Ellipse (in between)
  - General Shape
- **Stresses**
  - Hertzian
  - Non-hertzian
    - friction
    - plasticity
- **Applications**
  - Macroscale
    - bearings, gears, wheel/rail
  - Microscale
    - asperity interaction, wear

Hertzian contacts



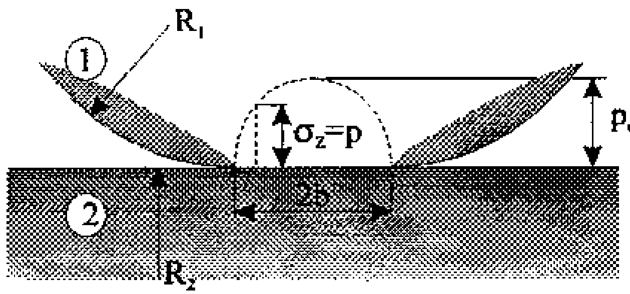
- Semi-infinite bodies
  - area of contact small compared to size of the body and radii of curvature
- The surfaces are continuous and non-conforming
- Frictionless

Hertzian Line Contact

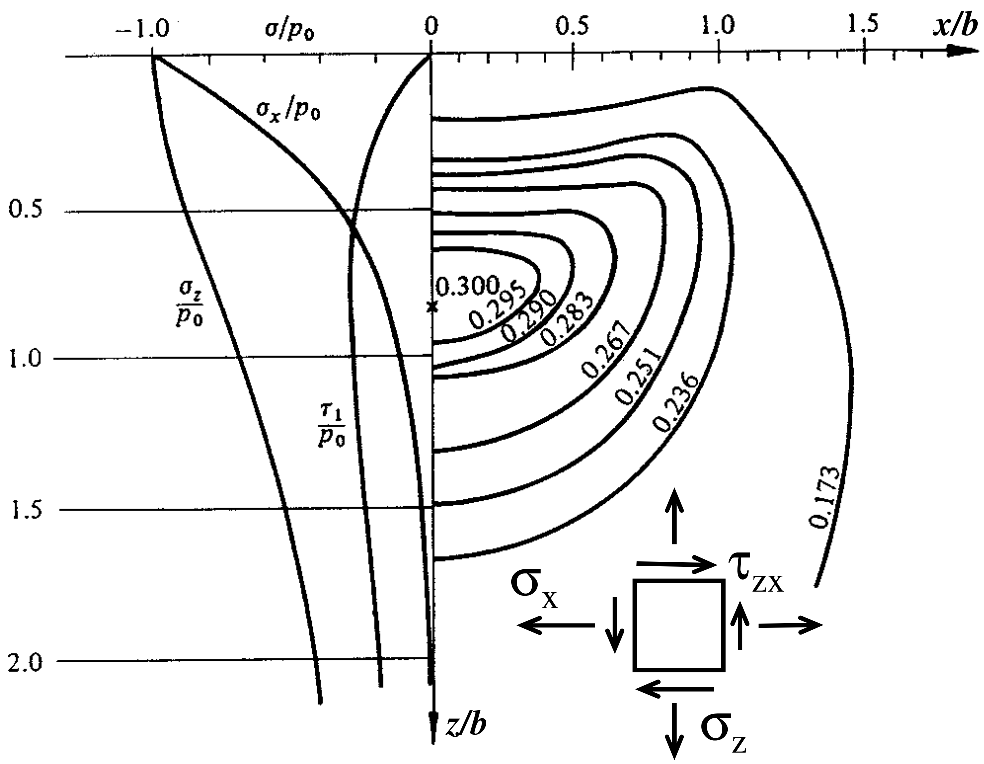


$$P_o = \left[\frac{P' E^*}{\pi R} \right]^{1/2}$$

P'=P/t = load per unit length
R = $(1/R_1+1/R_2)^{-1}$ = effective radius
E=combined elastic modulus*



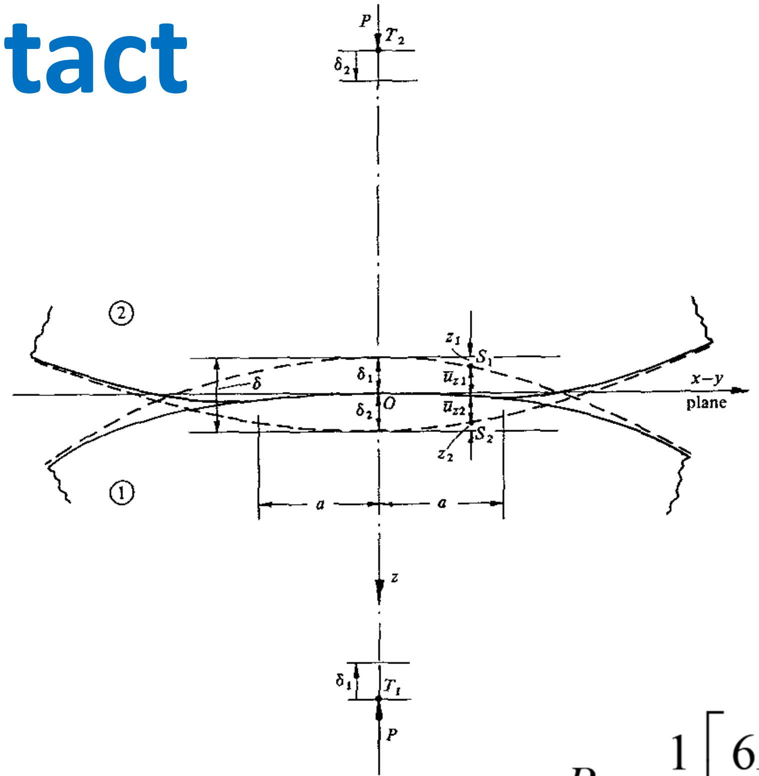
Line Contact Stress Field



The stress field appears the same in any parallel plane, i.e. "plane stress"

Point Contact

- e.g.
  - sphere on flat
  - sphere on sphere
  - two cylinders crossed at right angles

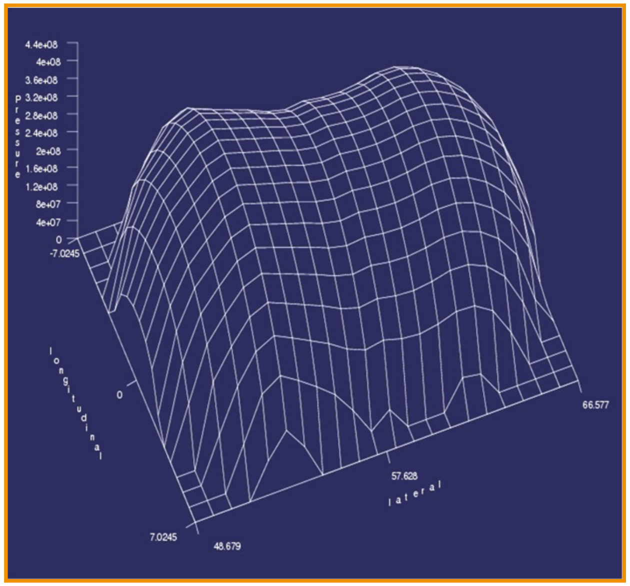


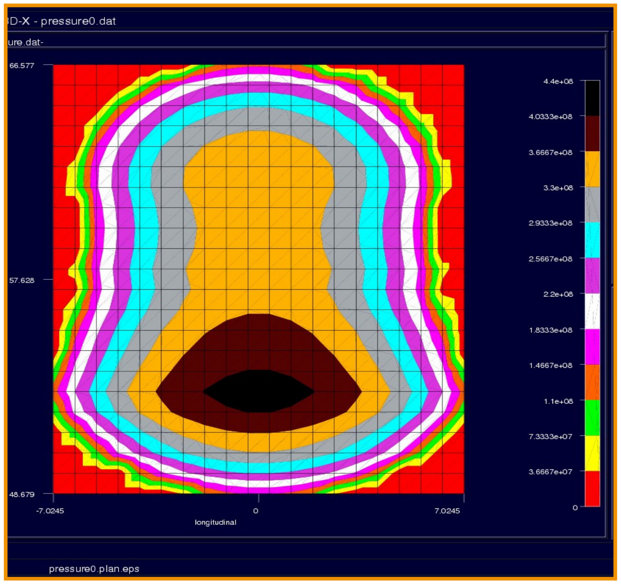
$$P_o = \frac{1}{\pi}\left[\frac{6PE^{*2}}{R'^2}\right]^{1/3}$$

Hertzian Formulae

	Line Contact Width 2b, Load P' per unit length	Circular Contact (diameter 2a, load P)
Semi-contact width or contact radius	$b = 2\left[\frac{P' R}{\pi E^*}\right]^{1/2}$	$a = \left[\frac{3}{4}\frac{PR}{E^*}\right]^{1/3}$
Maximum contact pressure ("Hertz Stress")	$P_o = \left[\frac{P' E^*}{\pi R}\right]^{1/2}$	$P_o = \frac{1}{\pi}\left[\frac{6PE^{*2}}{R^2}\right]^{1/3}$
Approach of centers	$\delta = \frac{2P'}{\pi}\left\{\frac{1-\nu_1^2}{E_1}\left[\ln\frac{4R_1}{b} - \frac{1}{2}\right] + \frac{1-\nu_2^2}{E_2}\left[\ln\frac{4R_2}{b}\right] - \frac{1}{2}\right\}$	$\delta = \frac{a^2}{R} = \frac{1}{2}\left[\frac{9}{2}\frac{P^2}{RE^{*2}}\right]^{1/3}$
Mean contact pressure	$\bar{p} = \frac{P'}{2b} = \frac{\pi}{4}P_o$	$\bar{p} = \frac{P}{\pi a^2} = \frac{2}{3}P_o$
Maximum shear stress	$\tau_{max} \cong 0.30\,P_o$ at (x=0, z=0.78b)	$\tau_{max} \cong 0.31\,P_o$ at (r=0, z=0.48a)

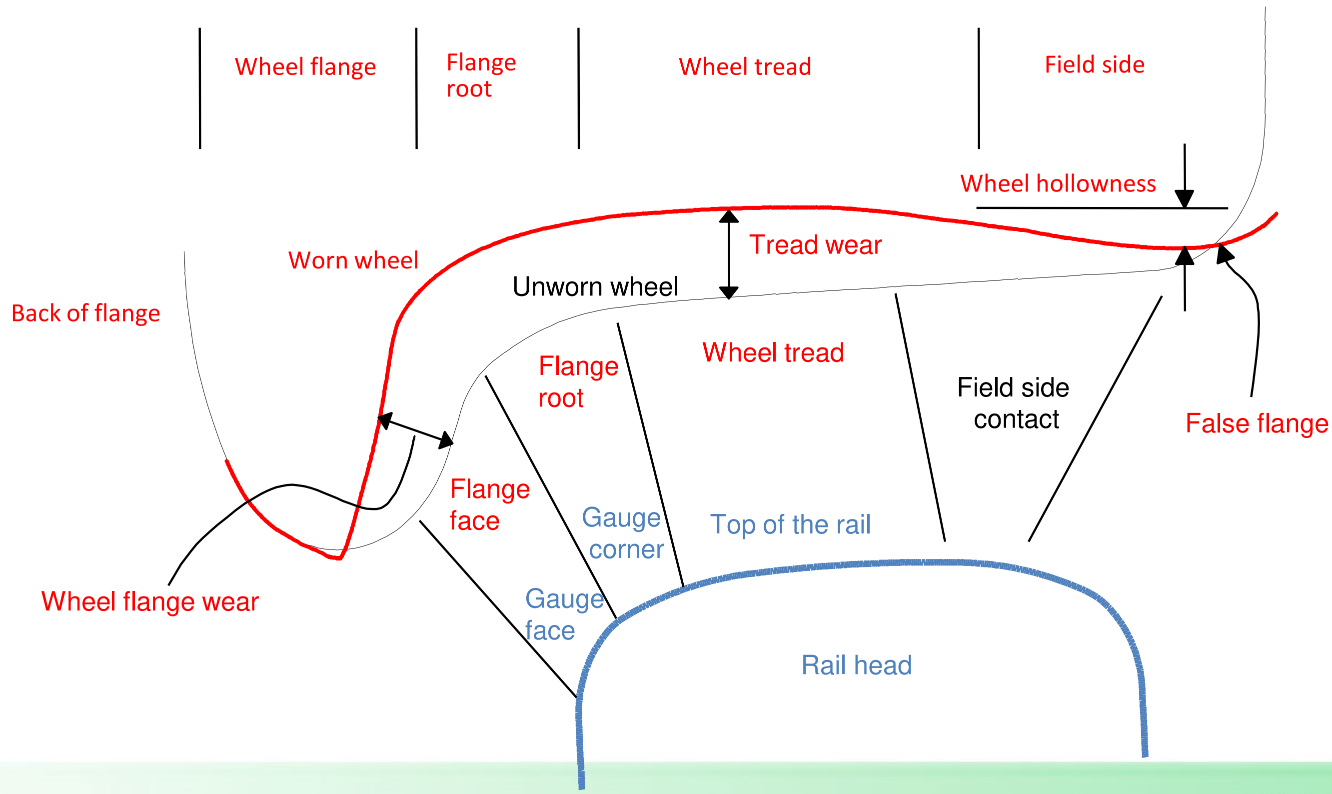
Most contacts are non-Hertzian





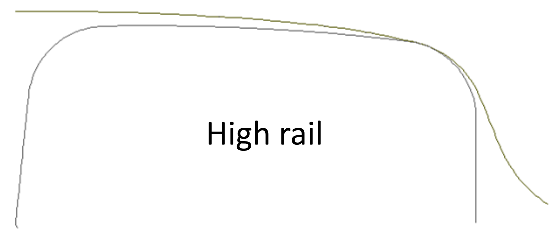
Generally: Hertzian assumption is not too bad: ±20%

Terminology

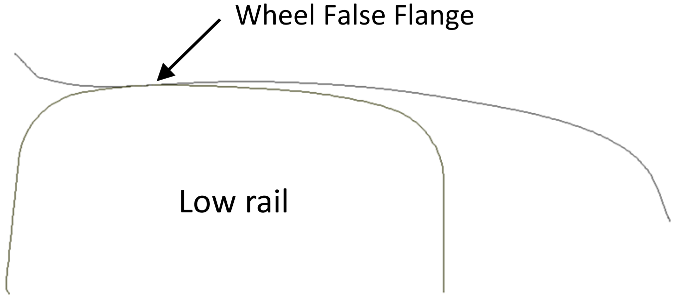


Wheel/rail stresses

- Stress and damage depend on:
  - wheel radius
  - wheel load
  - friction coefficient
  - wheel/rail profiles (contact geometry)

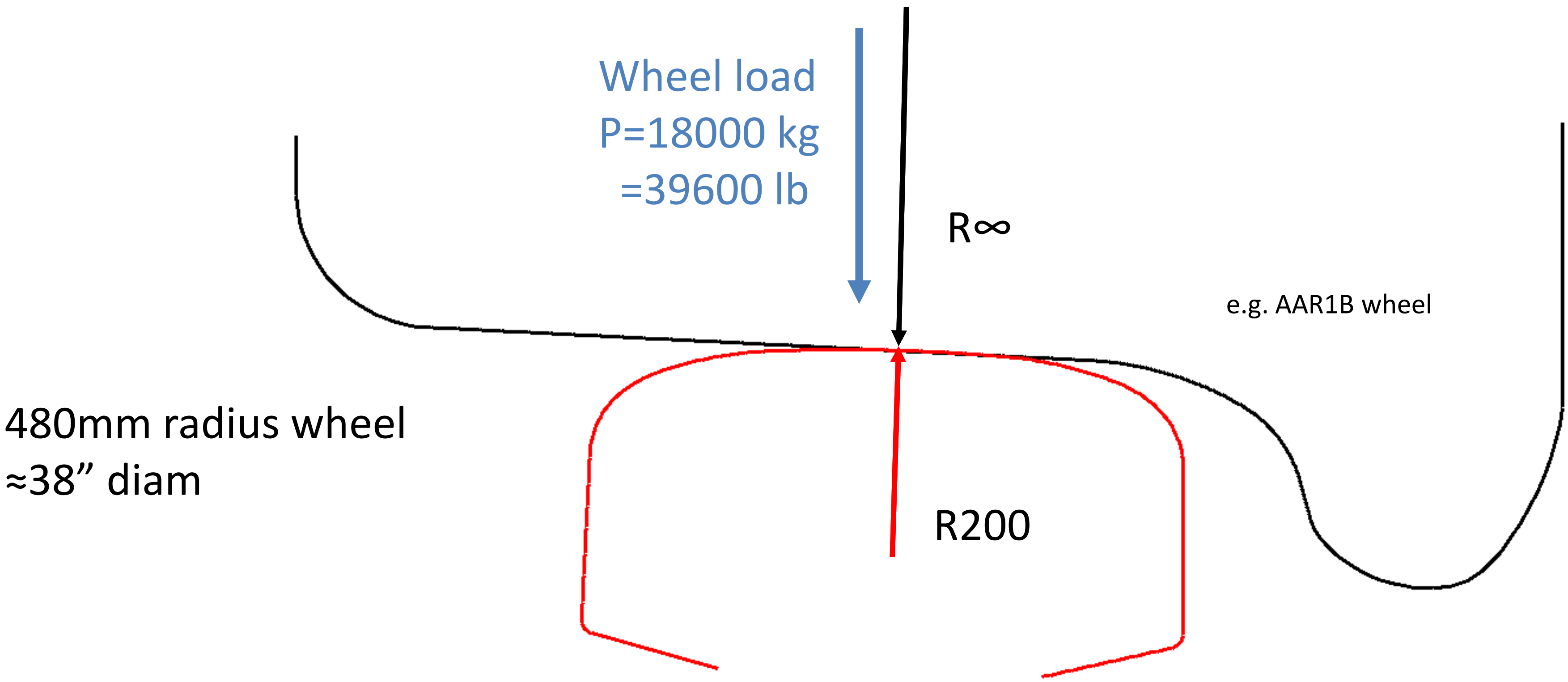


Non-conformal contact



Hollow wheels

Contact Stress Calculation #1



Contact stress calc. - TOR

- Steel wheel on Steel rail $E^* = \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}\right)^{-1}$

$v_1 = v_2 = 0.29, \quad E_1 = E_2 = 200\ GPa \quad \rightarrow \quad E^* = 109\ e9\ Pa = 1.58e7\ psi$

- 8” (200mm) rail head radius $\mathbf{R_{RT}} = 0.200\text{m}, \mathbf{R_{RL}} = \infty$
- New tapered wheel profile $\mathbf{R_{WT}} = \infty$
- Wheel radius is 480mm (≈19”) $\mathbf{R_{WL}} = 0.480\text{m}$
- Wheel load is 18000 kg X 9.81 ≈176.6 kN = **P**

Complete calculation

$$R_T = \left(\frac{1}{R_{RT}} + \frac{1}{R_{WT}}\right)^{-1} = \left(\frac{1}{0.20} + \frac{1}{\infty}\right)^{-1} = 0.20\text{m}$$

$$R_L = \left(\frac{1}{R_{RL}} + \frac{1}{R_{WL}}\right)^{-1} = \left(\frac{1}{\infty} + \frac{1}{0.480}\right)^{-1} = 0.480\text{m}$$

Elliptical contact

$$P_o = \frac{1}{\pi}\left[\frac{6PE^{*2}}{R^2}\right]^{1/3} F_1(R_L/R_T)$$

≈1.0

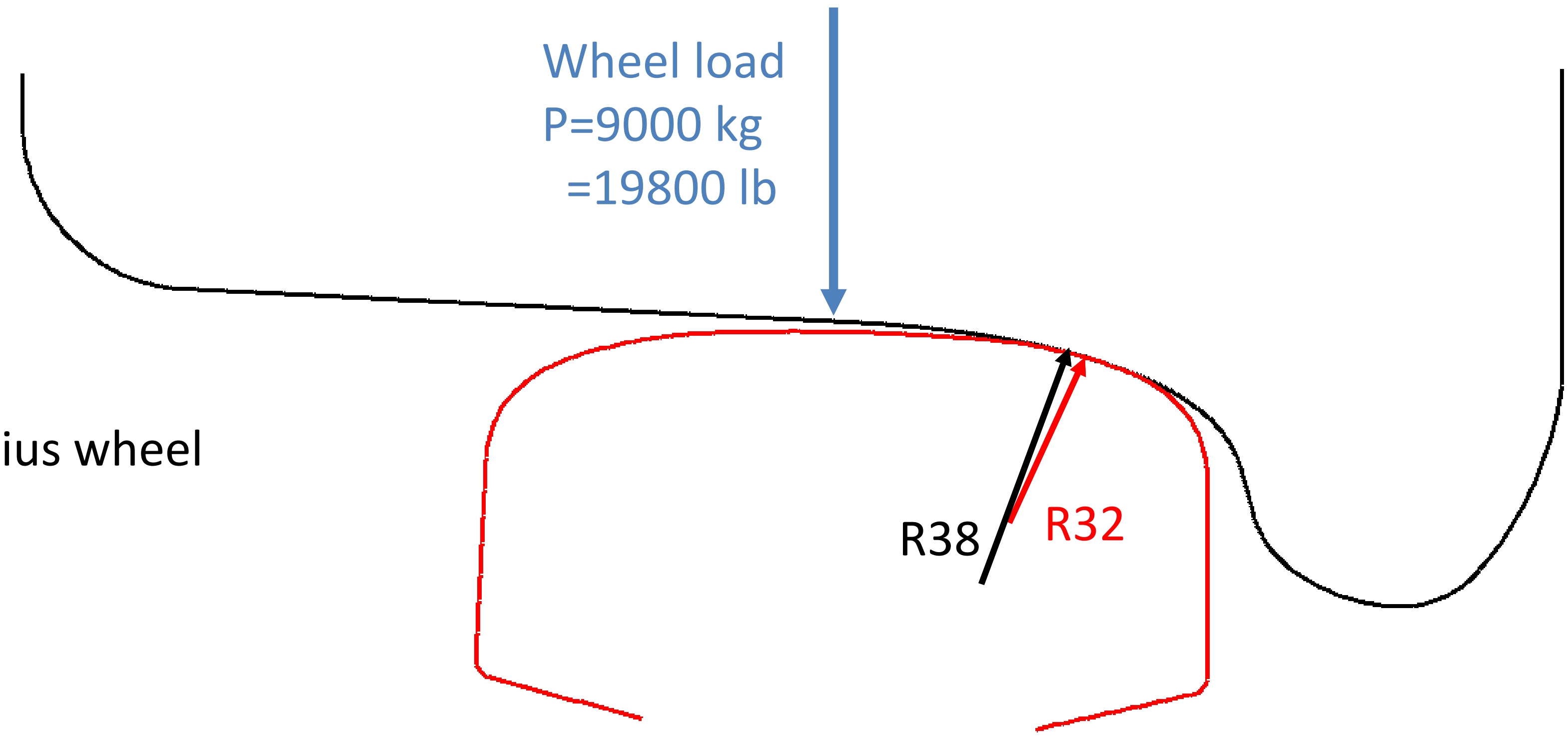
$$R = \sqrt{R_T R_L} = \sqrt{0.20 \times 0.480} = 0.31$$

$$a = \left[\frac{3}{4}\frac{PR}{E^*}\right]^{1/3} F_1(R_L/R_T)$$

$$P_O = \frac{1}{\pi}\left[\frac{6 \times 176{,}600 \times (109e^9)^2}{0.31^2}\right]^{1/3} = 1616\, e^6 \text{ Pa}$$

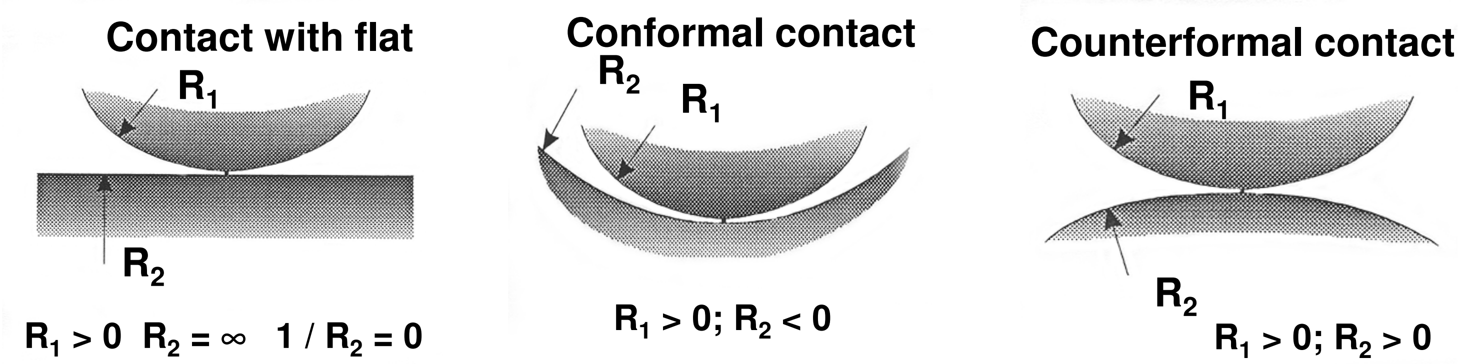
$$a = \left[\frac{3}{4}\,\frac{176600 \times 0.31}{109e^9}\right]^{1/3}$$ = 0.00722 m ≡ 7.22mm →14.5mm diam, 9/16”

Contact Stress calc. – gauge shoulder



240mm radius wheel
≈19” diam

Hertzian Contacts – Sign Convention



Contact stress calc. – rail shoulder

- 32 mm radius $\mathbf{R_{RT}} = 0.032\text{m},\ \mathbf{R_{RL}} = \infty$
- 38 mm flange root radius $\mathbf{R_{WT}} = -0.038\text{m}$
- Wheel radius is 240 mm $\mathbf{R_{WL}} = 0.240\text{m}$

$$\mathbf{R_L} = 0.240\text{m}$$

$$\mathbf{R_T} = \left(\frac{1}{0.032} - \frac{1}{0.038}\right)^{-1} = 0.2027$$

$$R = \sqrt{0.240 \times 0.2027} = 0.221$$

Complete calculation

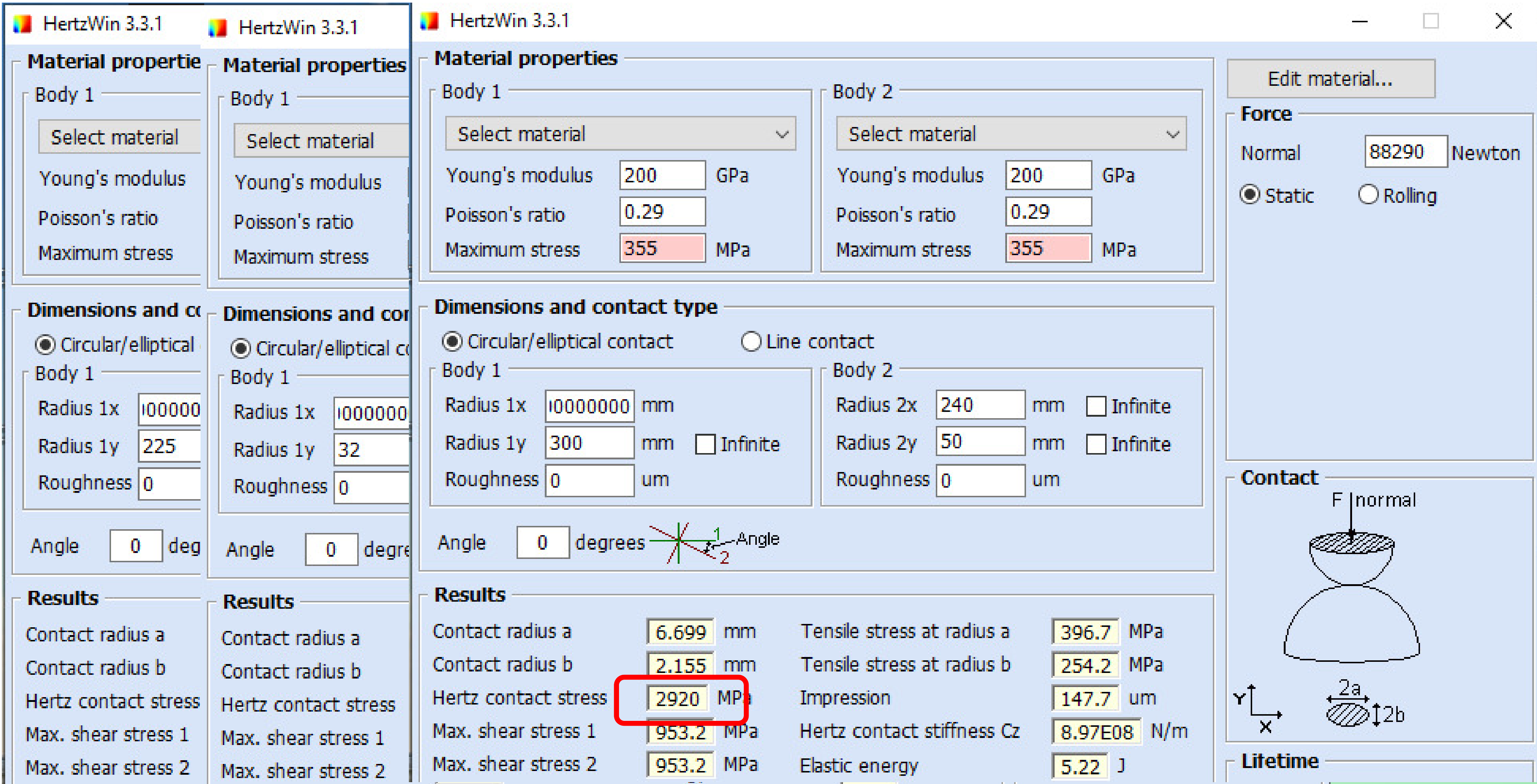
$$P = 9000 \times 9.81 = 88290N$$

$$P_o = \frac{1}{\pi}\left[\frac{6PE^{*2}}{R^2}\right]^{1/3}$$

$$P_O = \frac{1}{\pi}\left[\frac{6 \times 88290 \times \left(109e^9\right)^2}{0.221^2}\right]^{1/3} = 1608\ e^6 \text{ Pa}$$

$$a = \left[\frac{3}{4}\frac{PR}{E^*}\right]^{1/3}$$

$$a = \left[\frac{3}{4}\ \frac{88290 \times 0.221}{109e^9}\right]^{1/3}$$ = 0.00512 m ≡ 5.12mm →10.24mm diam, 13/32


HertzWin 3.3.1
Material properties
Body 1
Select material
Young's modulus 200 GPa
Poisson's ratio 0.29
Maximum stress 355 MPa
Body 2
Select material
Young's modulus 200 GPa
Poisson's ratio 0.29
Maximum stress 355 MPa
Edit material...
Force
Normal 88290 Newton
Static
Rolling
Dimensions and contact type
Circular/elliptical contact
Line contact
Body 1
Radius 1x 10000000 mm
Radius 1y 300 mm Infinite
Roughness 0 um
Body 2
Radius 2x 240 mm Infinite
Radius 2y 50 mm Infinite
Roughness 0 um
Angle 0 degrees
Angle
Contact
F normal
2a
2b
Results
Contact radius a 6.699 mm
Contact radius b 2.155 mm
Hertz contact stress 2920 MPa
Max. shear stress 1 953.2 MPa
Max. shear stress 2 953.2 MPa
Tensile stress at radius a 396.7 MPa
Tensile stress at radius b 254.2 MPa
Impression 147.7 um
Hertz contact stiffness Cz 8.97E08 N/m
Elastic energy 5.22 J
Lifetime
Radius 1y 225
Radius 1y 32

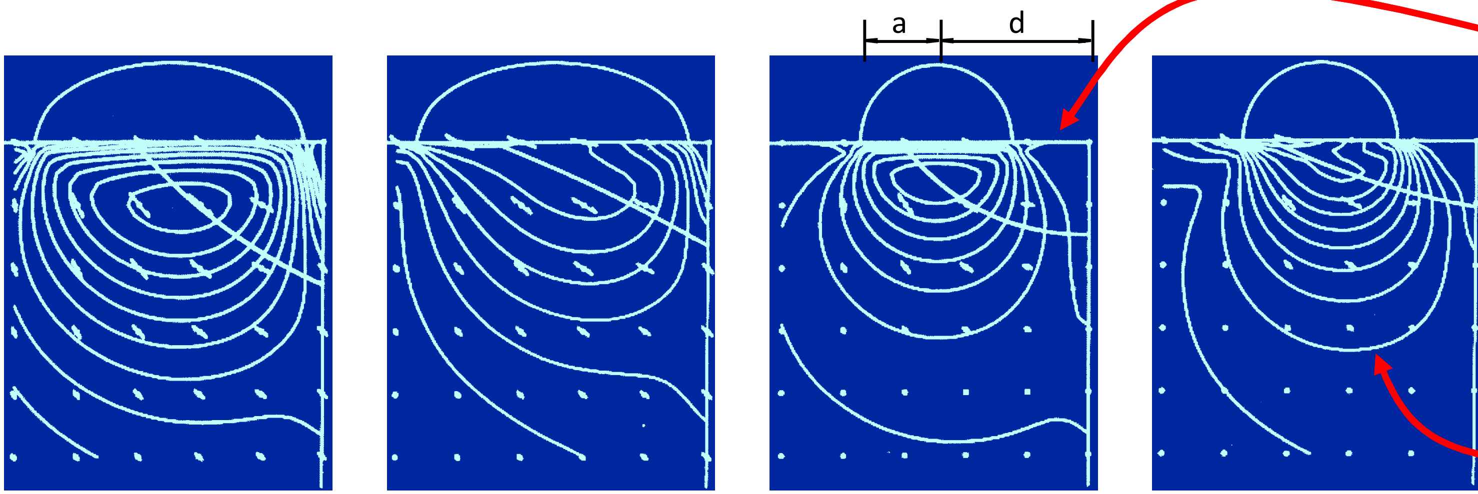
Rail/Wheel: Hertzian Contact Stress (MPa)

$$P_o = \left(\frac{6PE^{*2}}{\pi^3 R_e^2} \right)^{1/3} \times \left[\; F_1 (R_L / R_T)^{-2/3} \; \right]$$

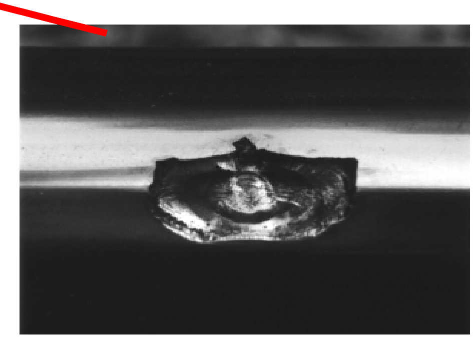
spherical contacts — accounts for ellipticity

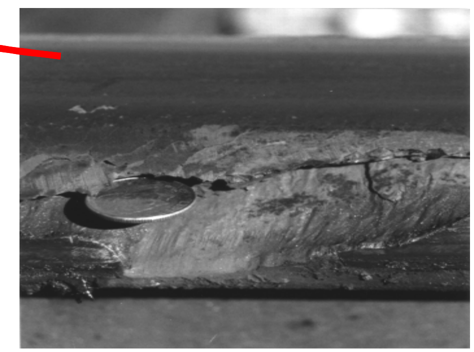
Location	Traverse Radius Rail (mm)	Traverse Radius Wheel (mm)	Load, Wheel Radius: 18 Tonnes 480 mm		18 Tonnes 240 mm		9 Tonnes 240 mm	
Rail Crown	+200	-300	1130	(1.00)	1438	(1.27)	1141	(1.01)
	+75	-100	1428	(1.26)	1794	(1.59)	1424	(1.26)
	+100	-300	1819	(1.61)	2267	(2.01)	1800	(1.59)
	+200	infinity	1645	(1.46)	2053	(1.82)	1629	(1.44)
Rail Shoulder	+32	-38	1637	(1.45)	2043	(1.81)	1622	(1.44)
	+32	-44	1984	(1.76)	2469	(2.18)	1960	(1.73)
Flange Root	+8	-9.5	2678	(2.37)	3317	(2.94)	2632	(2.33)
False Flange	+300	+50	2845	(2.52)	3520	(3.12)	2794	(2.47)

Elastic loading of quarter space



d/a=1.14 T/N=0.0 | d/a=1.14 T/N=0.4 | d/a=2.0 T/N=0.0 | d/a=2.0 T/N=0.4

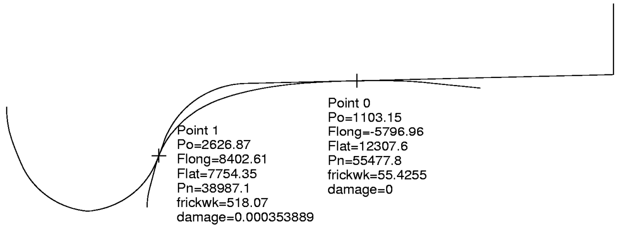


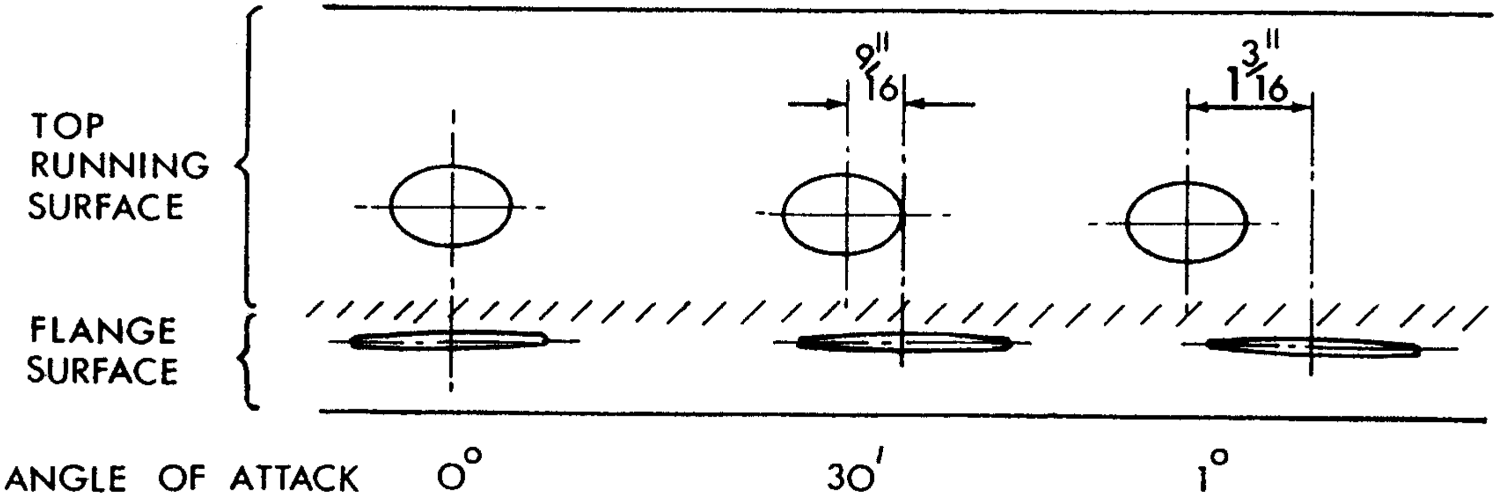


- **Leads to gauge corner collapse**
- **In lubricated track, the DSS's are "cylindrical"**
- **In dry track, the DSS's are "flatter" or "straight"**
- **Collapse is stronger if closer to the edge**

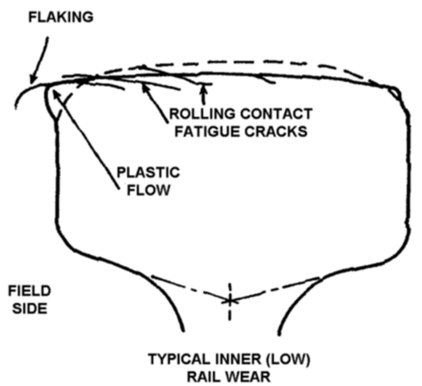
Wheel/rail contact

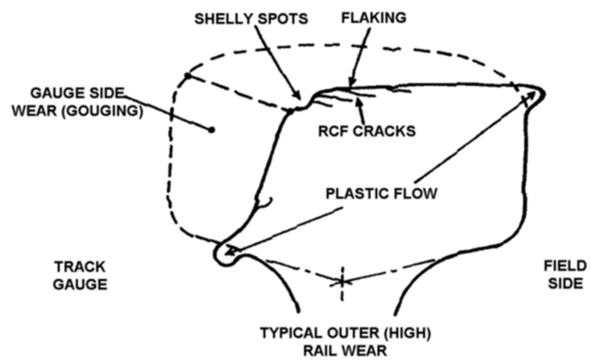
- Plan view of contact ellipses on high rail for different angles of attack

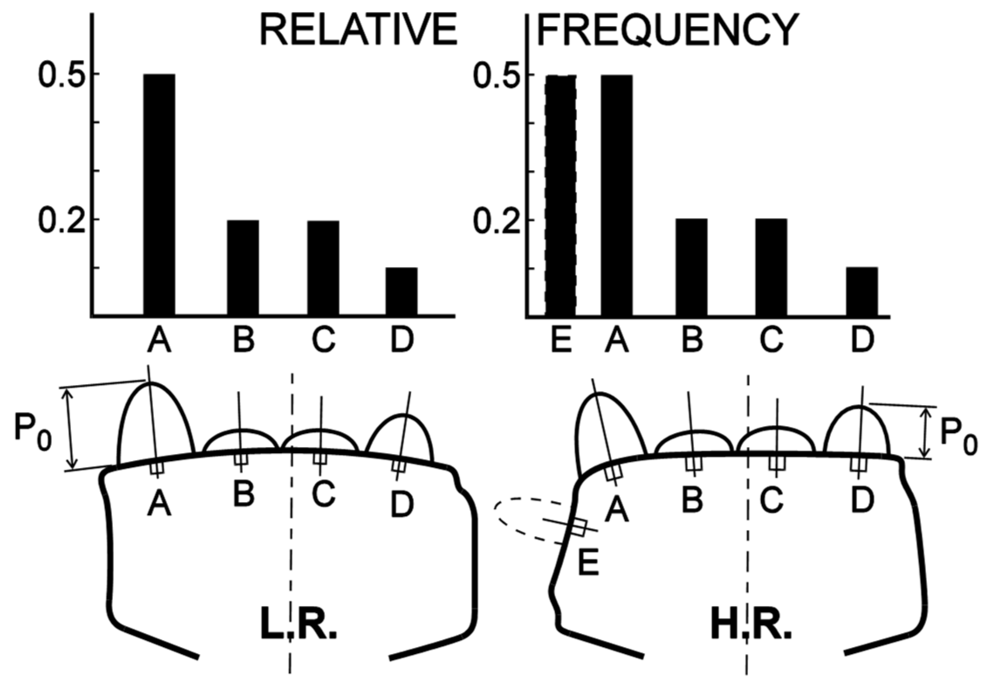




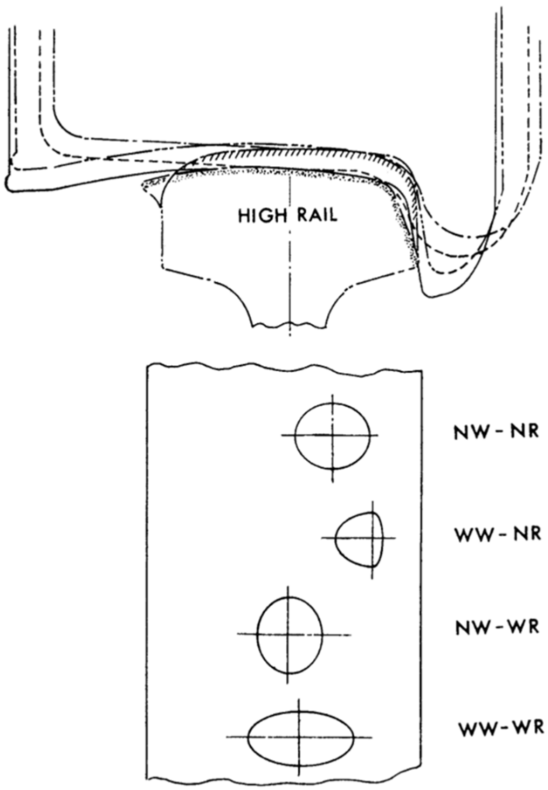
Surface Damage and Pummelling

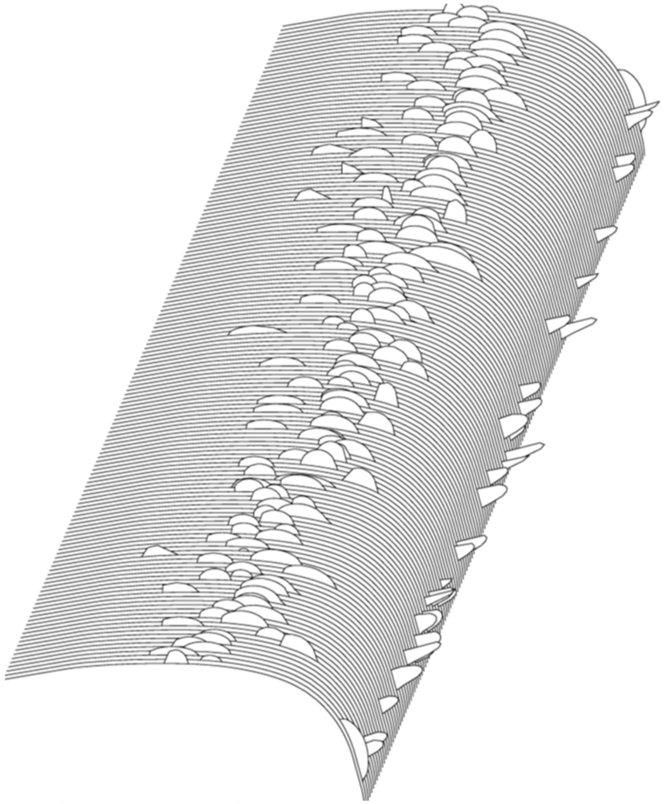


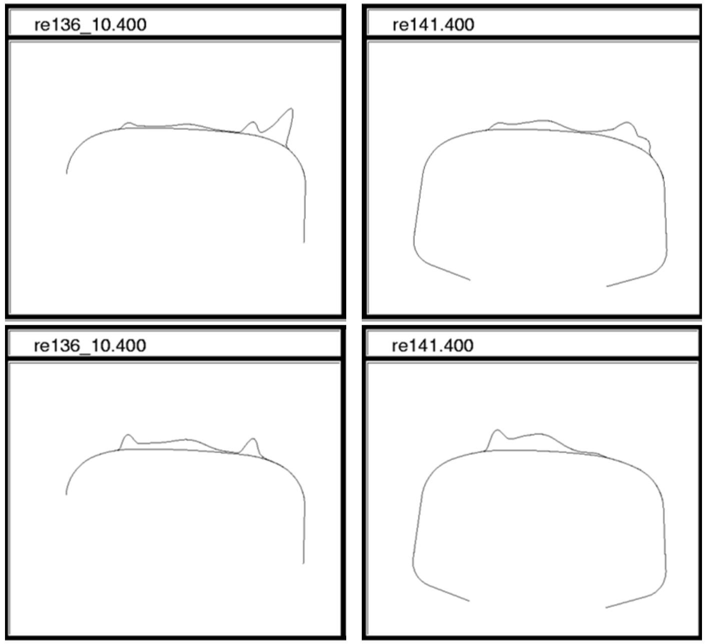




Pummelling


HIGH RAIL
NW - NR
WW - NR
NW - WR
WW - WR

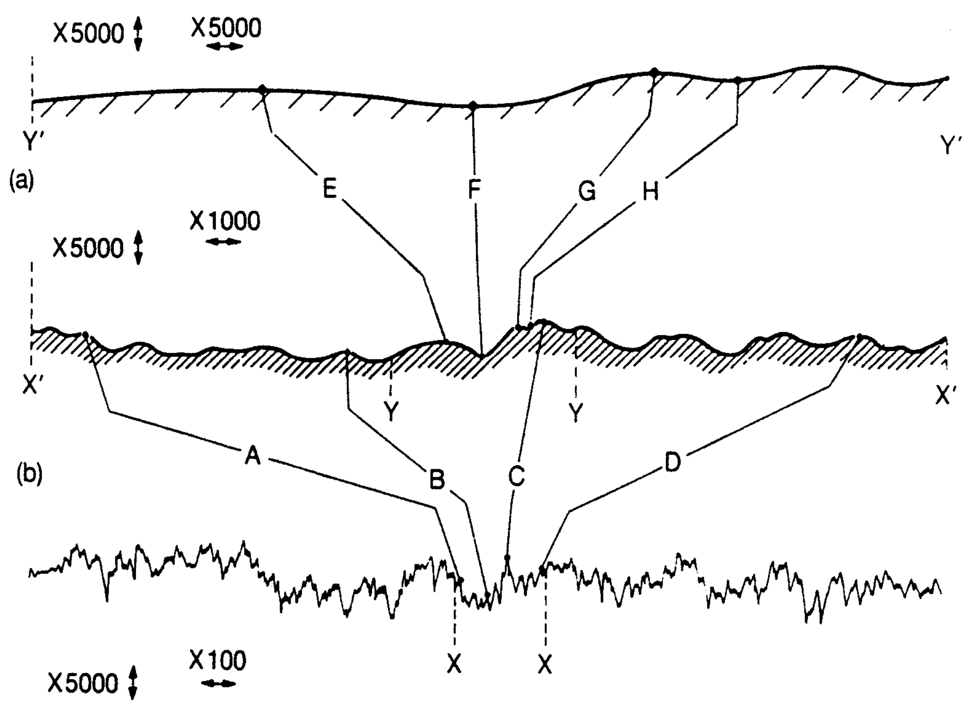



re136_10.400
re141.400
re136_10.400
re141.400

The influence of

SURFACE ROUGHNESS

Surface Roughness



On a micro-scale, all surfaces are rough

from Dagnall H, *Exploring Surface Texture*, Rank Taylor Hobson (1980).

Contact between real surfaces

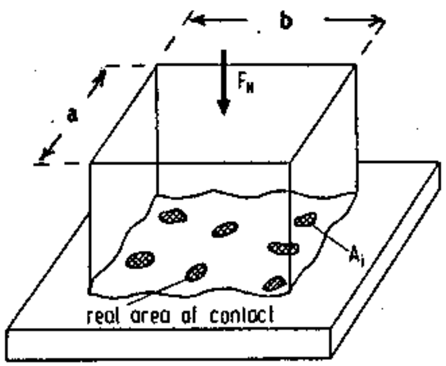
- Real area of contact is much smaller than the nominal area
- Apparent area:

$$A_A = ab$$

- Real area

$$A_R = \sum_{i=1}^{n} A_i$$

- Pressure = load/area



Contact Stress

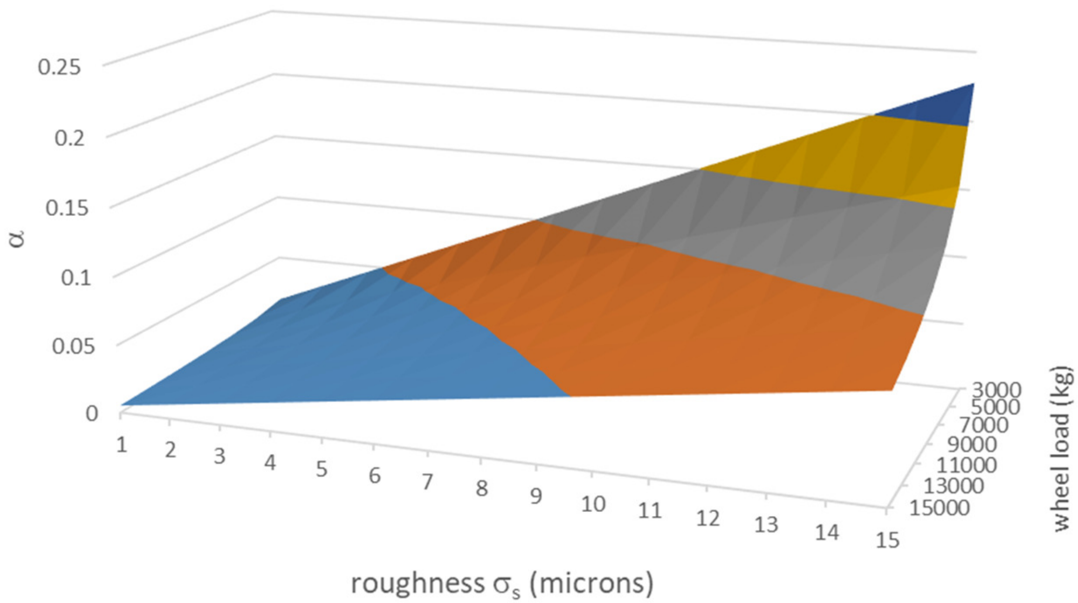
- Elastic contact models can be applied with errors of only a few percent if the combined roughness of the two surfaces is less than about 5% of the bulk elastic compression, i.e.

$$\alpha \equiv \frac{\sigma}{\delta} = \sigma \left(\frac{16E^*R}{9P} \right)^{1/3} < 0.05$$

KL Johnson,
Contact Mechanics
Section 13.5

- Hertzian spring: 0.05 – 0.15mm => 2.5 – 7.5 μm

Effect of wheel load and wheel radius



$\sigma = (\sigma_1{}^2 + \sigma_2{}^2)^{1/2}$

Where:

σ = composite roughness

σ_1 σ_2 = RMS roughness values of bodies 1 and 2

$\sigma_{RMS} = 1.3\ \sigma_{CLA}$ (measured)

200mm rail crown radius

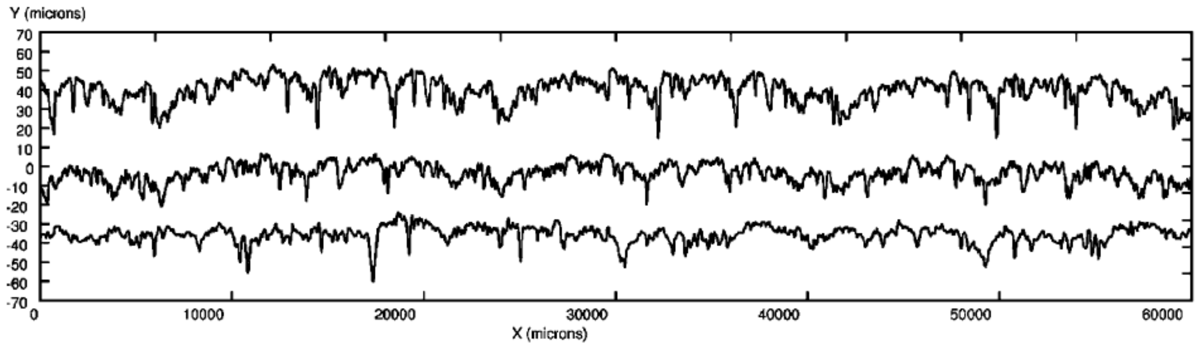
457 mm wheel radius (36" diam)

Cutting wheel diam in half increases roughness threshold by approx. 1 micron

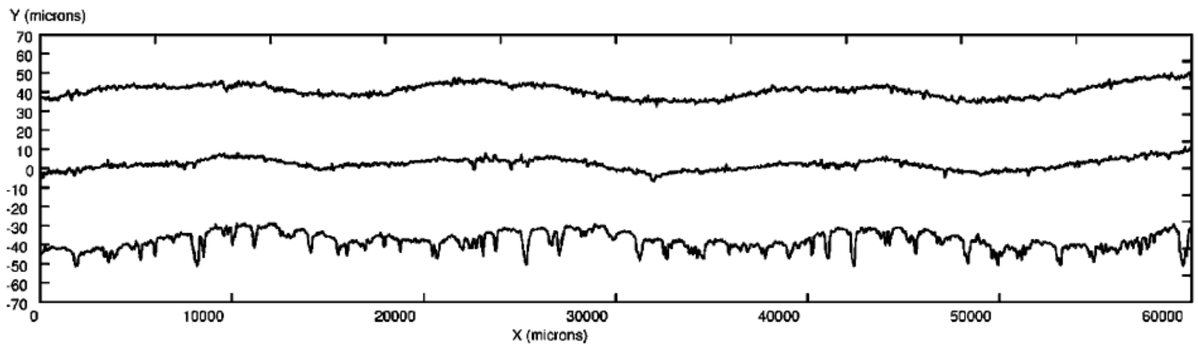
Roughness threshold (microns)

Wheel load		Wheel radius (in/m)	
klb	kg	18/0.457	9/0.229
6600	3000	3	3.5
19800	9000	7	7.5
33000	15000	9.5	10.5

Roughness from rail grinding



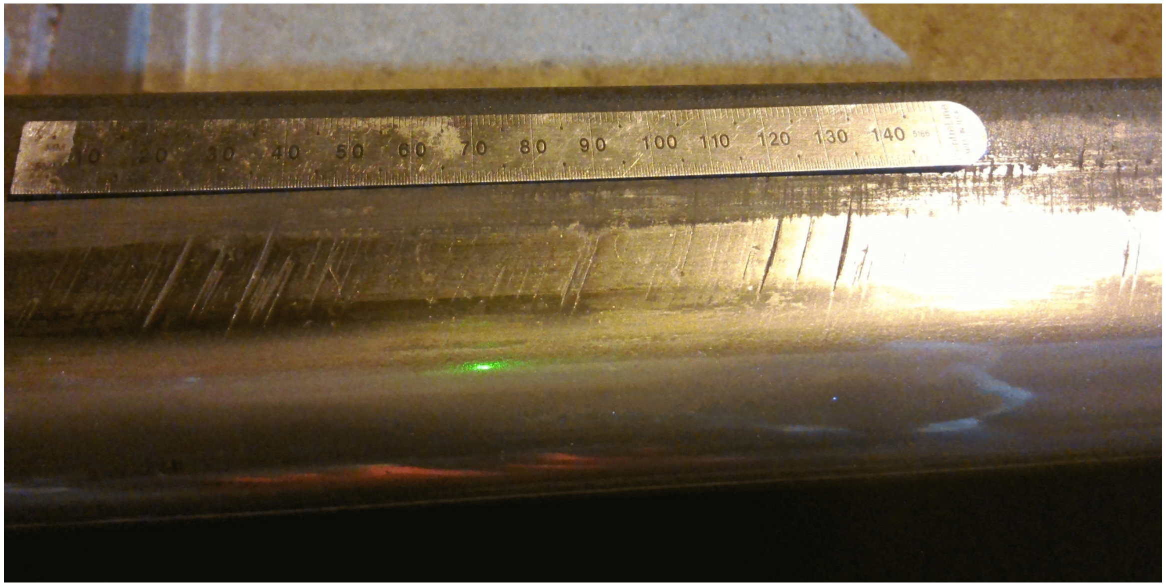
(A) Immediately after grinding



(B) Approx. 2MGT after grinding

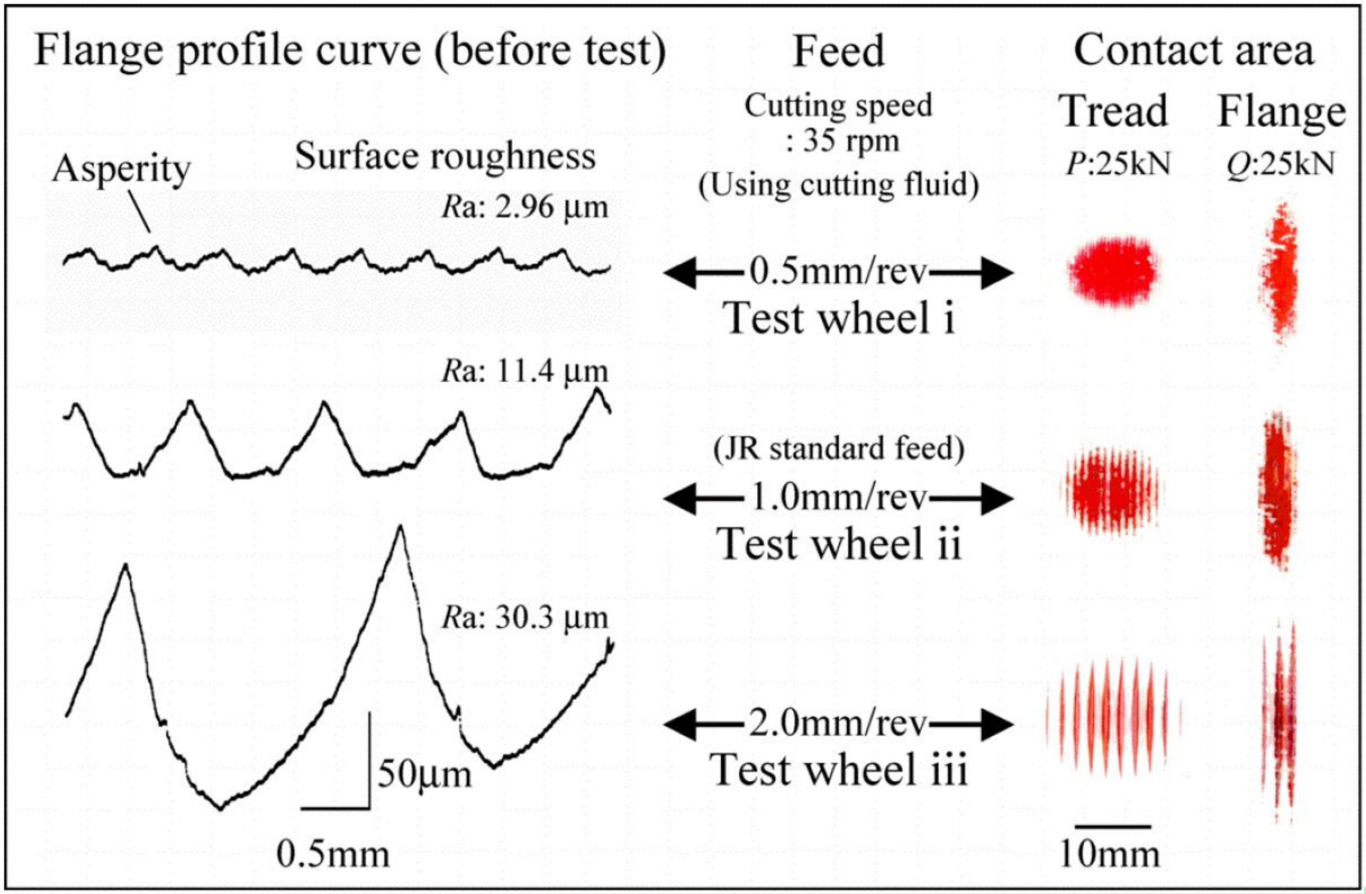
US Transit

“Rough” grinding



<u>May</u> contribute to noise and vibration, corrugation, RCF and squats/studs

The rough wheel and wheel climb



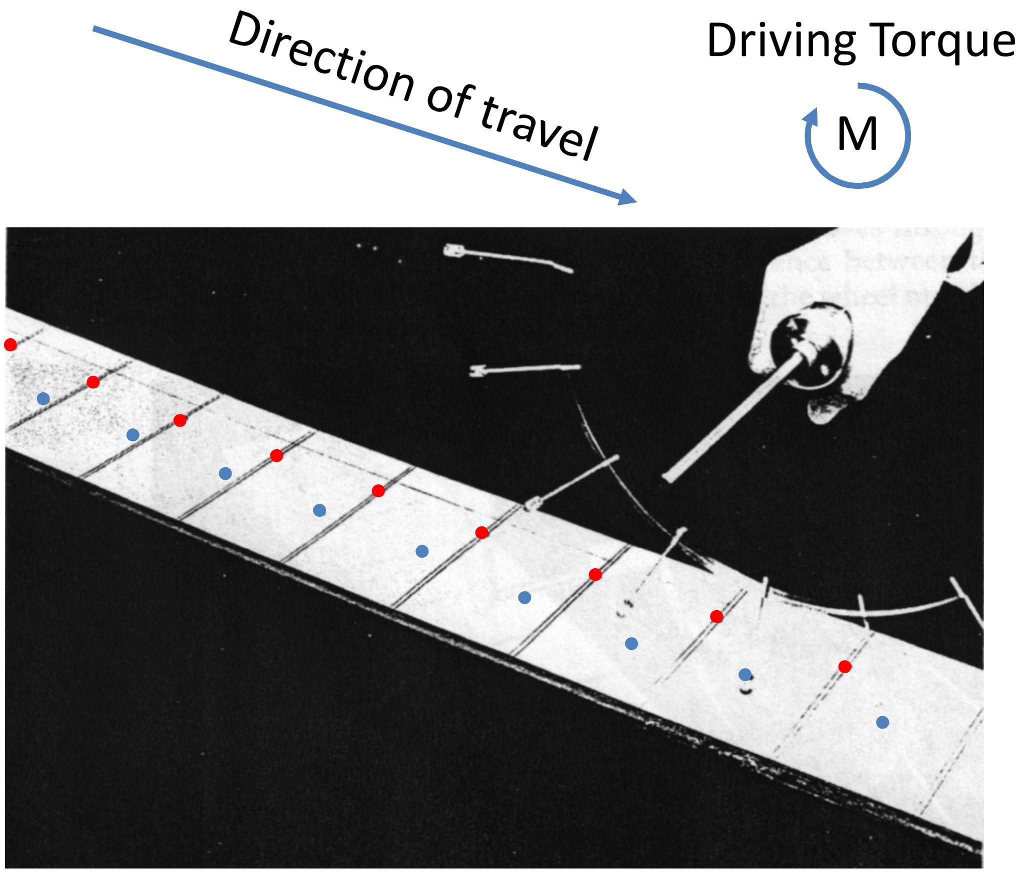
T. Ban et al, *A study on the coefficient of friction between rail gauge corner and wheel flange focussing on wheel machining*, Proceedings International Wheelset Congress, Orlando, 2004

Surface Roughness - conclusion

- Important
  - high frequency phenomena (noise, vibration)
  - Deformation of the micro-surface layer
- Little impact
  - bulk contact stresses
  - Wheel/rail forces
- Wheel roughness ⟷ wheel climb ??
- Rail corrugation: ± 30% on hertz stress

CREEPAGE/SLIP

Longitudinal Creepage

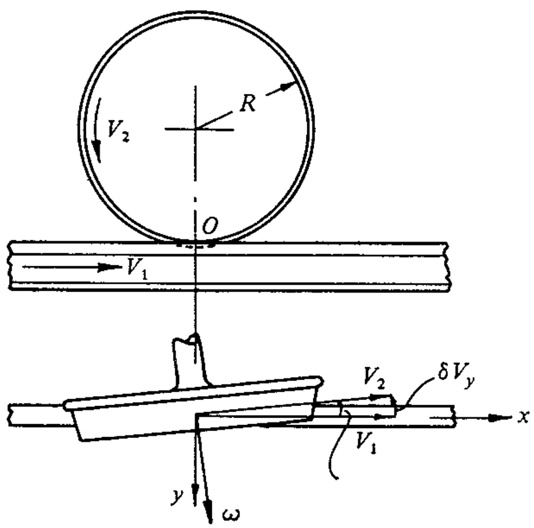


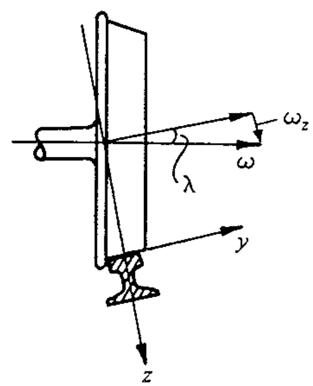
From AH Wickens (1978), Dynamics and the Advanced Passenger Train

1% creepage

Under traction: 1.01 revolutions of wheel to travel 1 circumference (e.g. 363.6° vs 360°)

Creepage in a single wheel/rail contact





Longitudinal Creepage

$$\psi_X = \frac{V_2 - V_1}{V_1}$$

Lateral Creepage

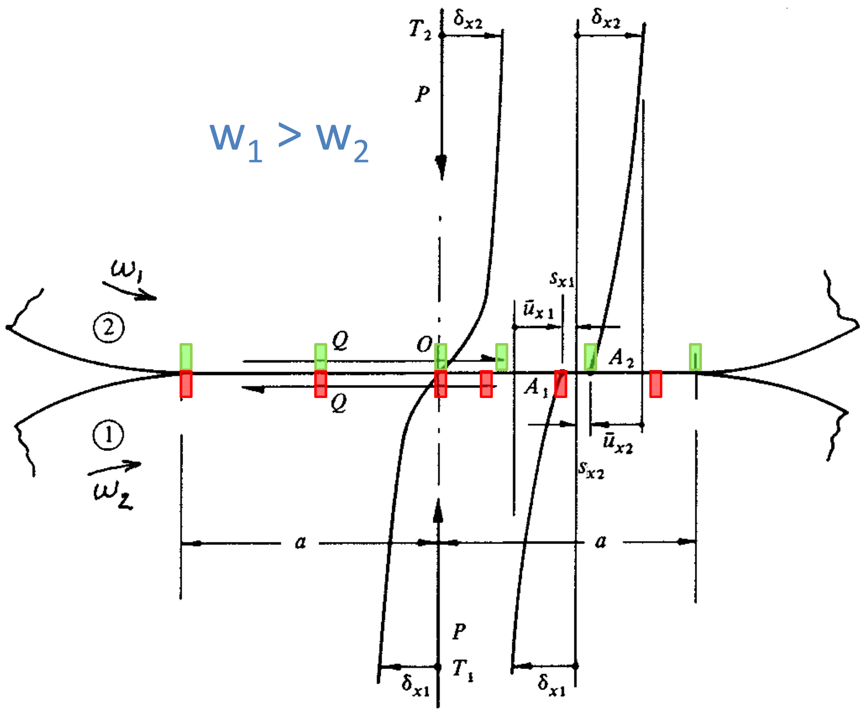
$$\psi_Y = \frac{\delta V_Y}{V_1} = \tan\gamma$$

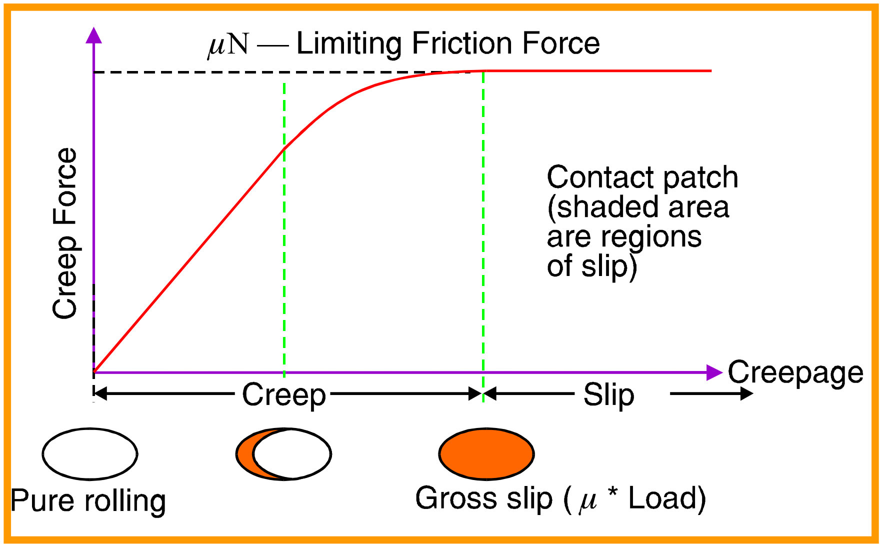
Spin Parameter

$$\Phi = \omega\frac{(ab)^{1/2}}{V_1 R} = \left(\frac{(ab)^{1/2}}{R}\right)\tan\lambda$$

Stick and Slip in the Contact Patch

From KL Johnson (1987), Contact Mechanics, Fig 7.6

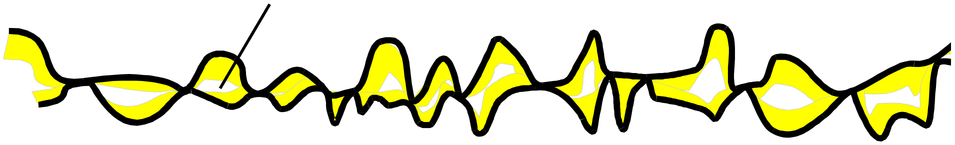




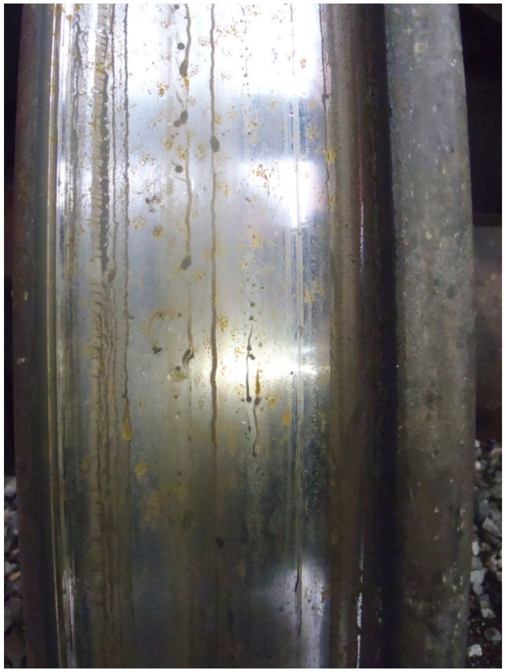
Elastic deformation in rolling bodies in stick and slip regions in rolling sliding contact

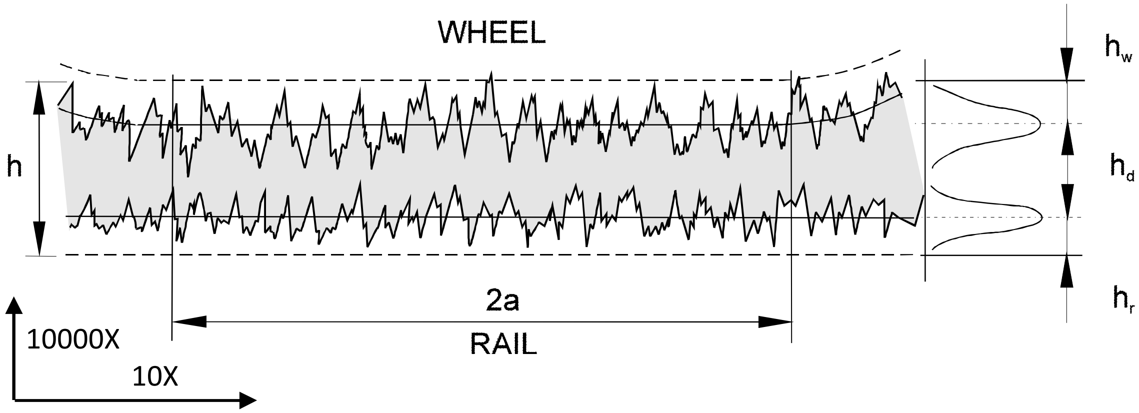
THIRDBODY LAYERS AND THE TRACTION-CREEP RELATIONSHIP

Third-body layer



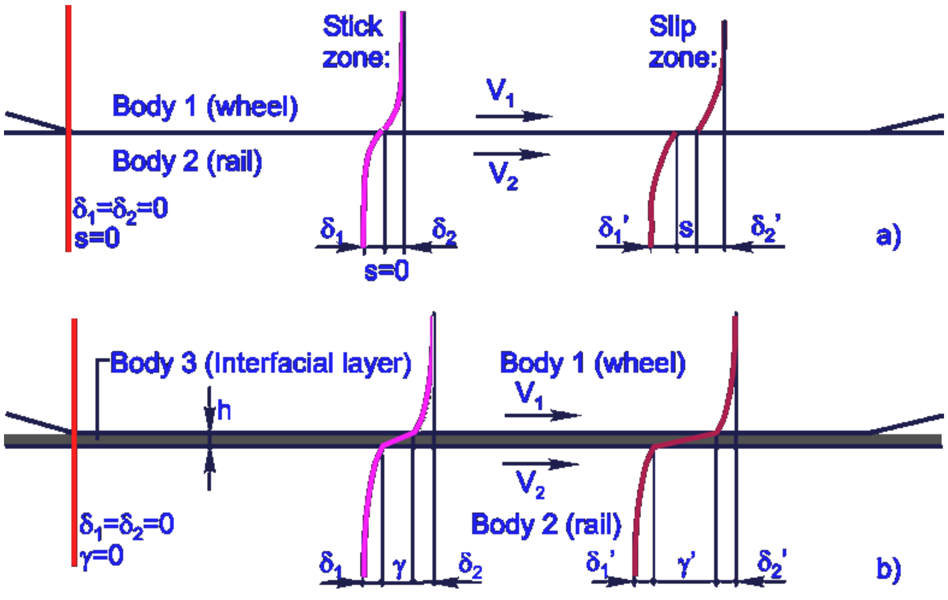
- Petrochemical:
oil, soap, grease
- Solid / mechanical:
moly, graphite
- Chemical:
phosphate, salts, etc.





LAYERS:
Any microscopic mixture of solid and semi-solid particles

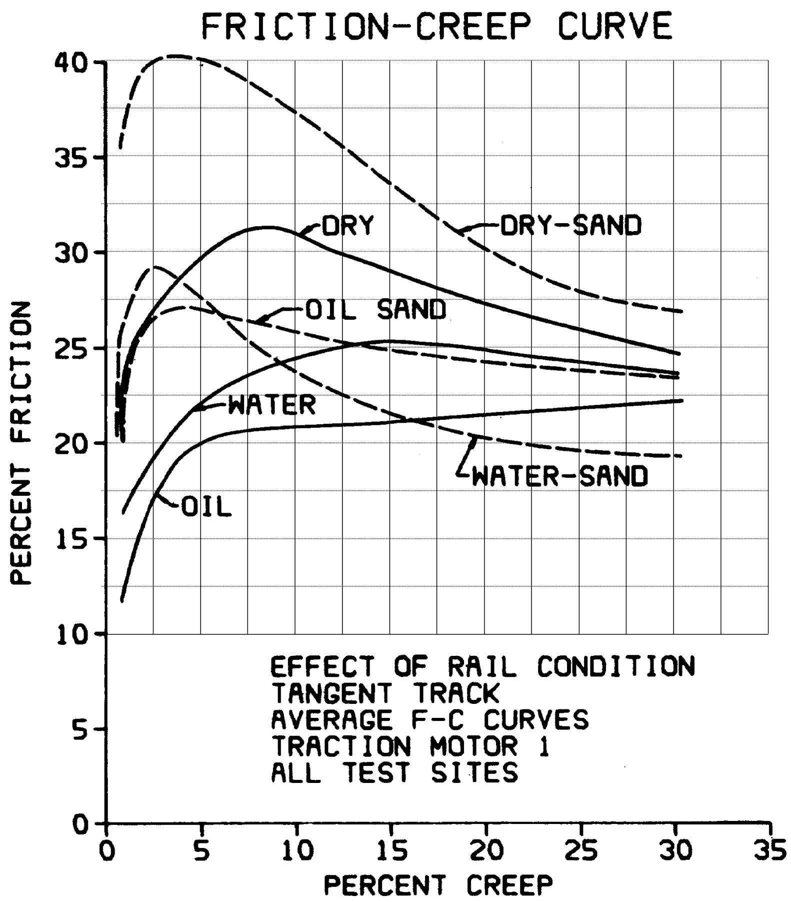
Thirdbody layer



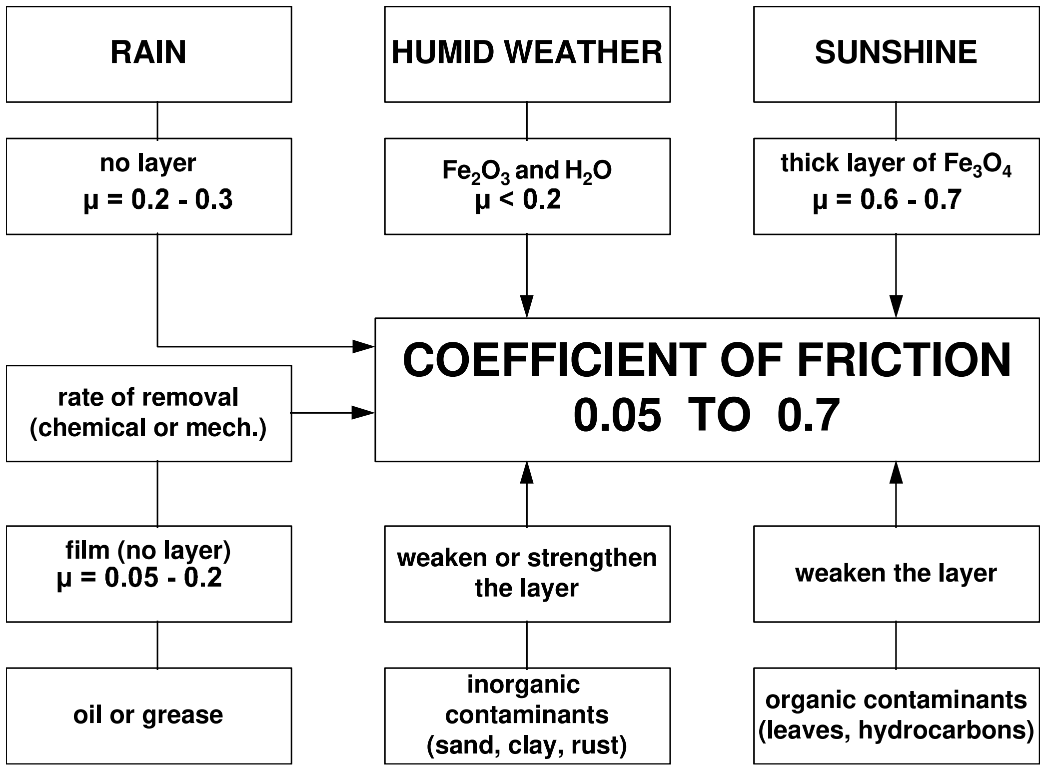
Wheel/rail traction-creepage curve

Field Tests

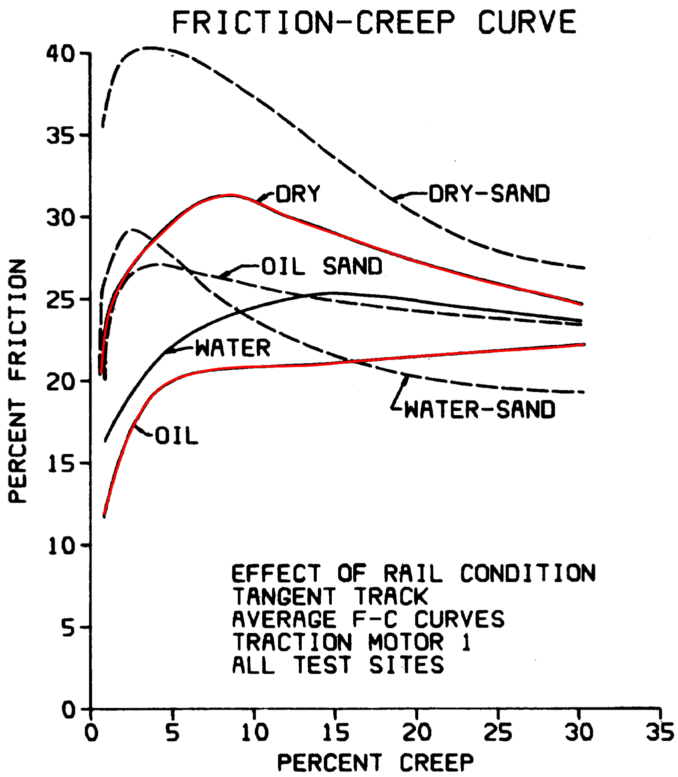
(Logston & Itami 1980)

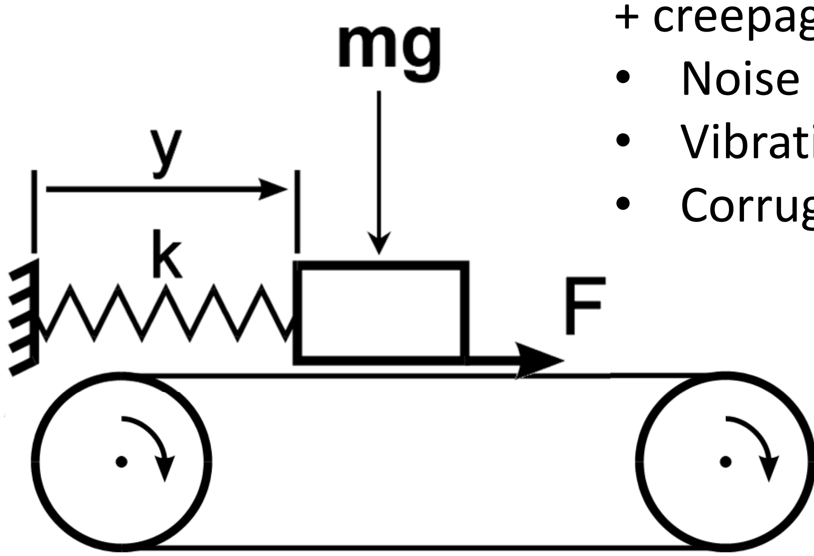


COF (μ) at TOR/wheel-tread contact



Stick-Slip - The prony brake



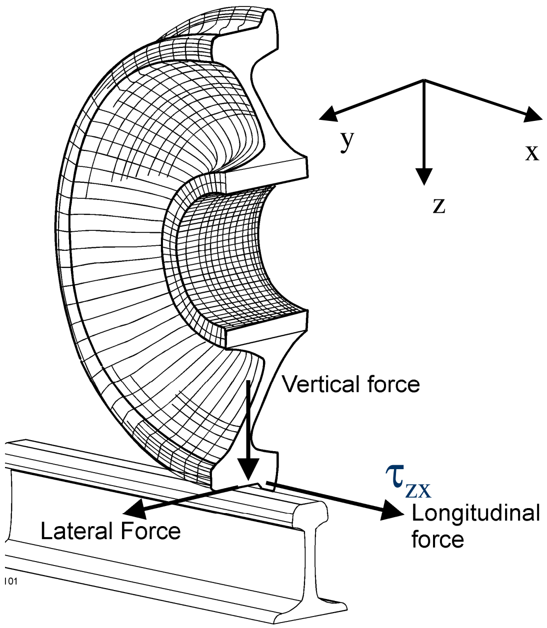


Negative Friction Characteristic + creepage

- Noise
- Vibration
- Corrugation

Wheel/rail stresses

- Vertical, longitudinal, and lateral forces
- Lead to a complex stress field
  - Compressive, tensile and shear stress components
- P_0 is maximum normal contact stress
- Important stresses = τ_{zx}, τ_{zy}
  - The stress on the z plane in the x and y direction
  - Cause shear of rail surface



Effect of shear stress

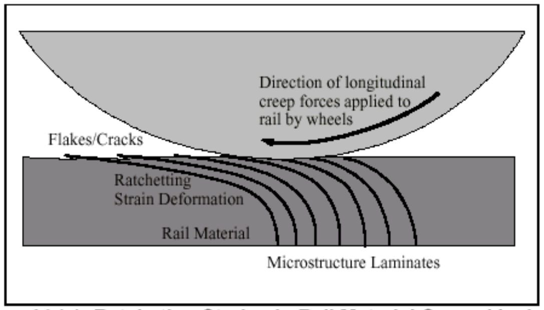
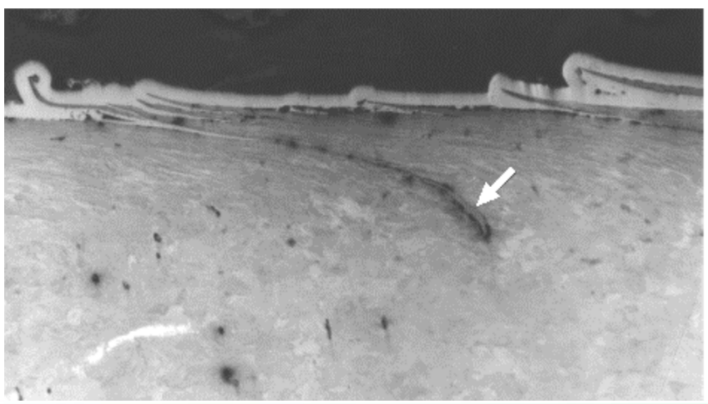
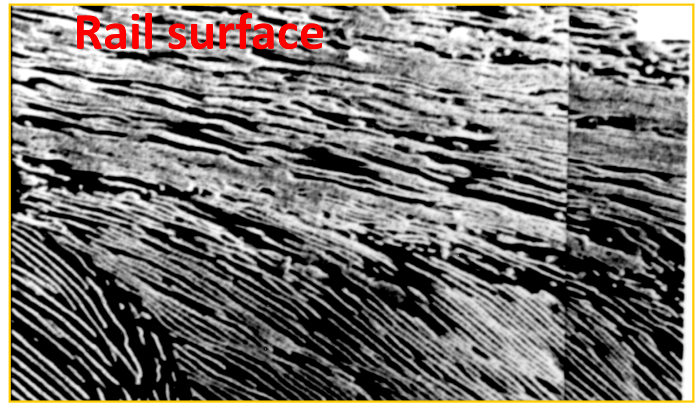
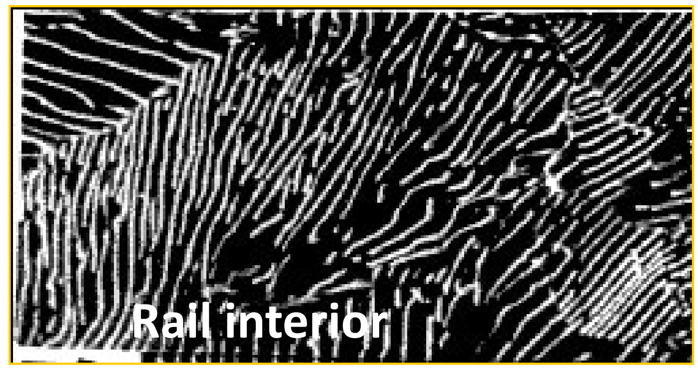


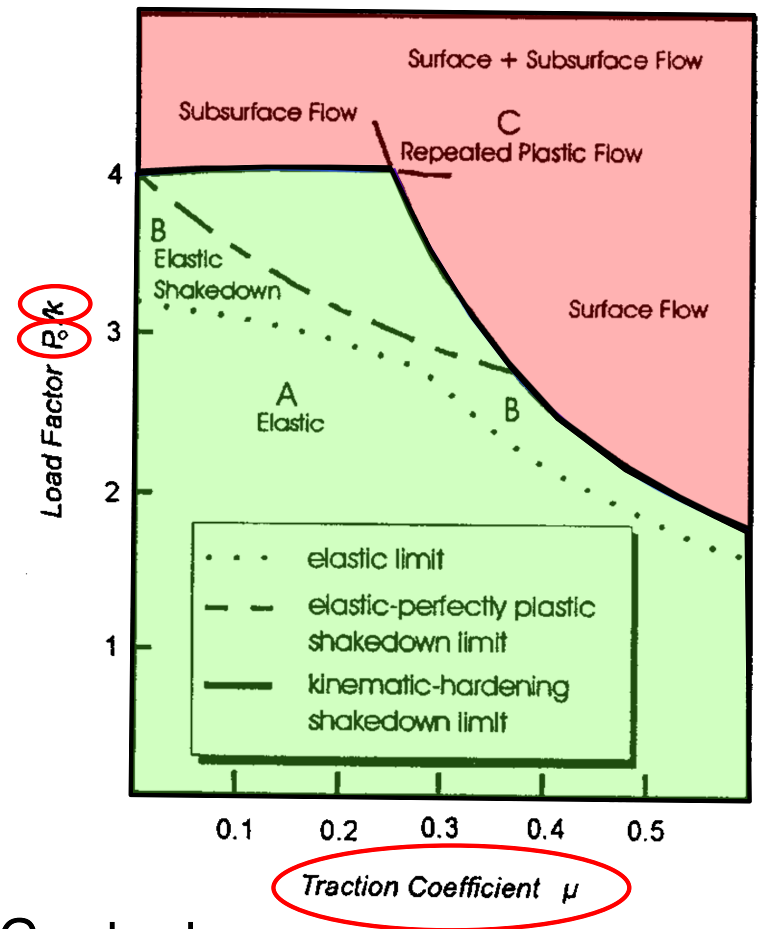
Figure 14.(c): Ratcheting Strains in Rail Material Caused by Large Longitudinal Creep Forces Between Wheel and Rail



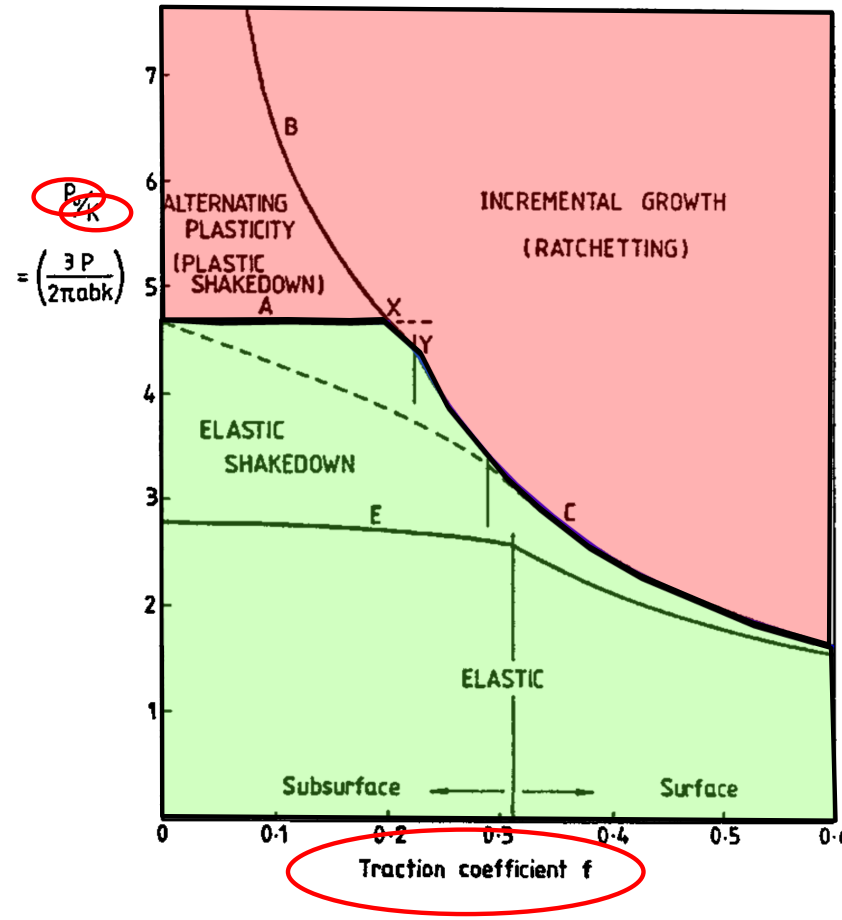




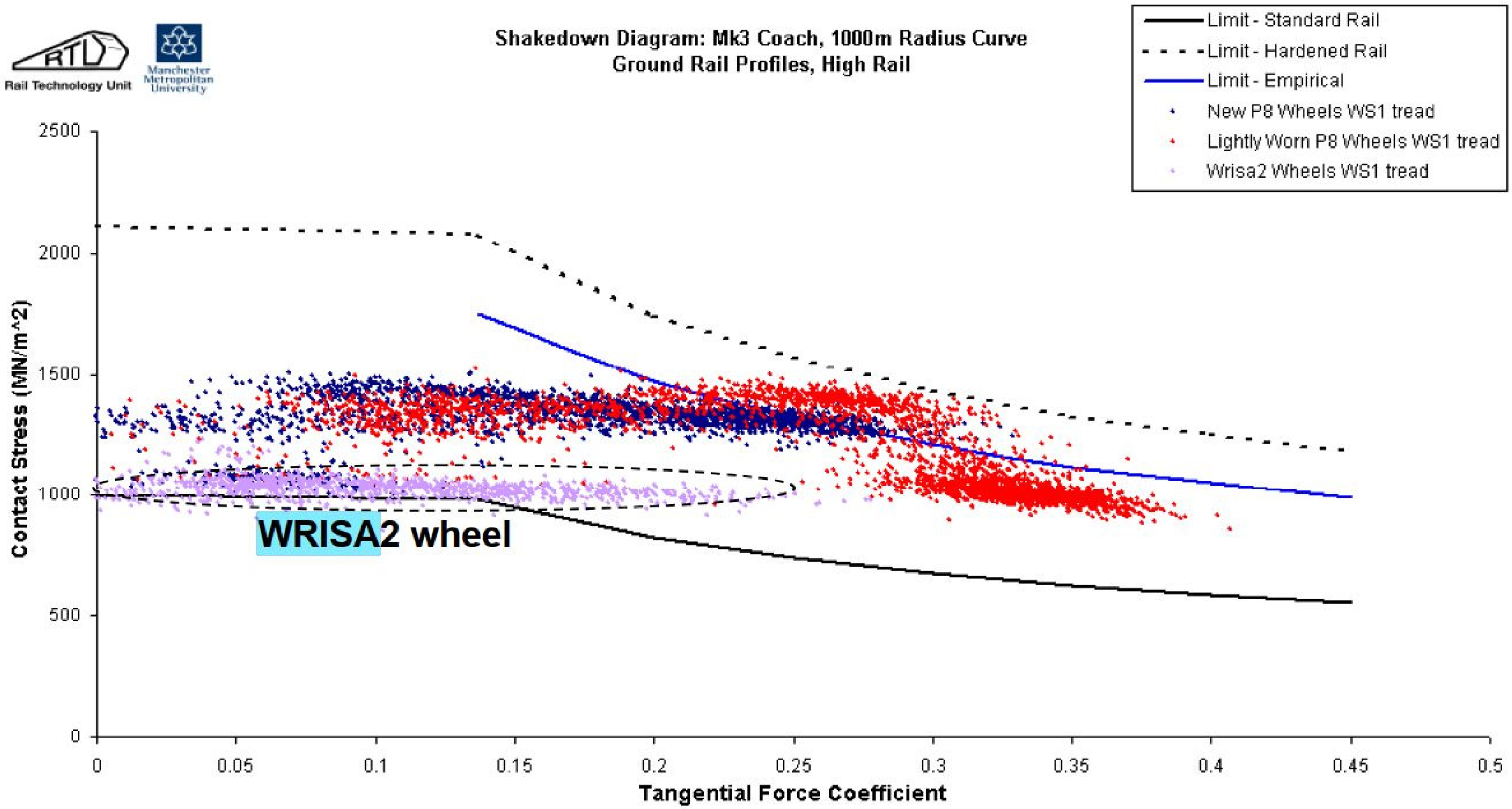
SHAKEDOWN



Line Contacts



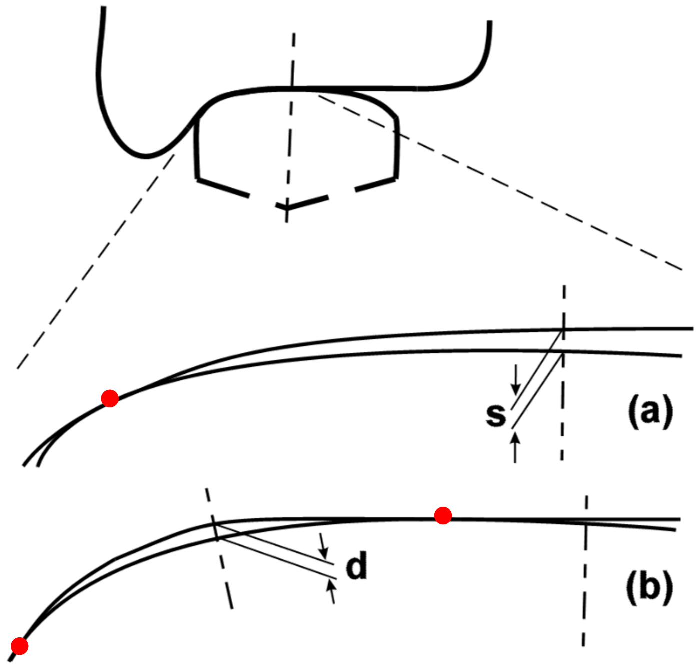
Point Contacts



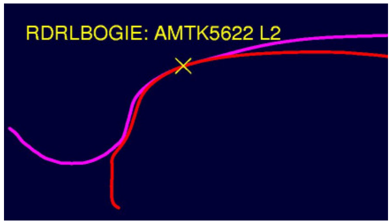
Rail Technology Unit
Manchester Metropolitan University
Shakedown Diagram: Mk3 Coach, 1000m Radius Curve
Ground Rail Profiles, High Rail
Limit - Standard Rail
Limit - Hardened Rail
Limit - Empirical
New P8 Wheels WS1 tread
Lightly Worn P8 Wheels WS1 tread
Wrisa2 Wheels WS1 tread
Contact Stress (MN/m^2)
Tangential Force Coefficient
2500
2000
1500
1000
500
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
WRISA2 wheel

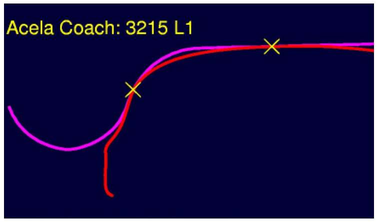
CONFORMALITY

Conformality

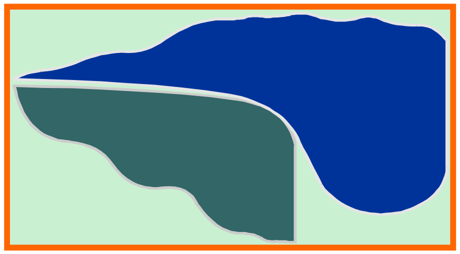


- closely conformal (as per hertzian spring)
  0.1 mm (0.004”) or less
- conformal
  0.1 mm to 0.4mm
  (0.004” to 0.016”)
- non-conformal
  0.4 mm (0.016”) or larger

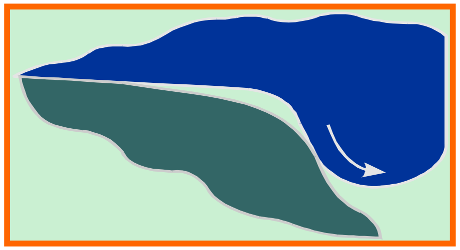




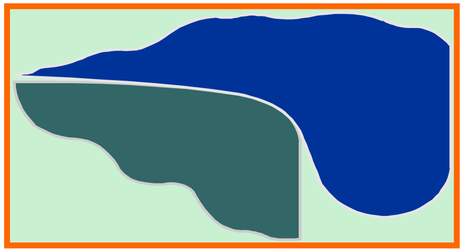
Conformality



Single Point Contact

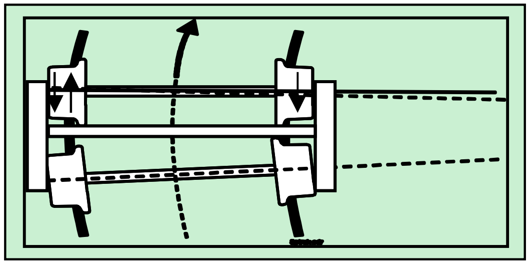


Two Point Contact

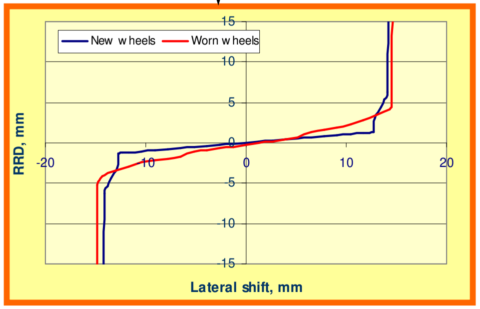


Conformal Contact

Gage corner damage

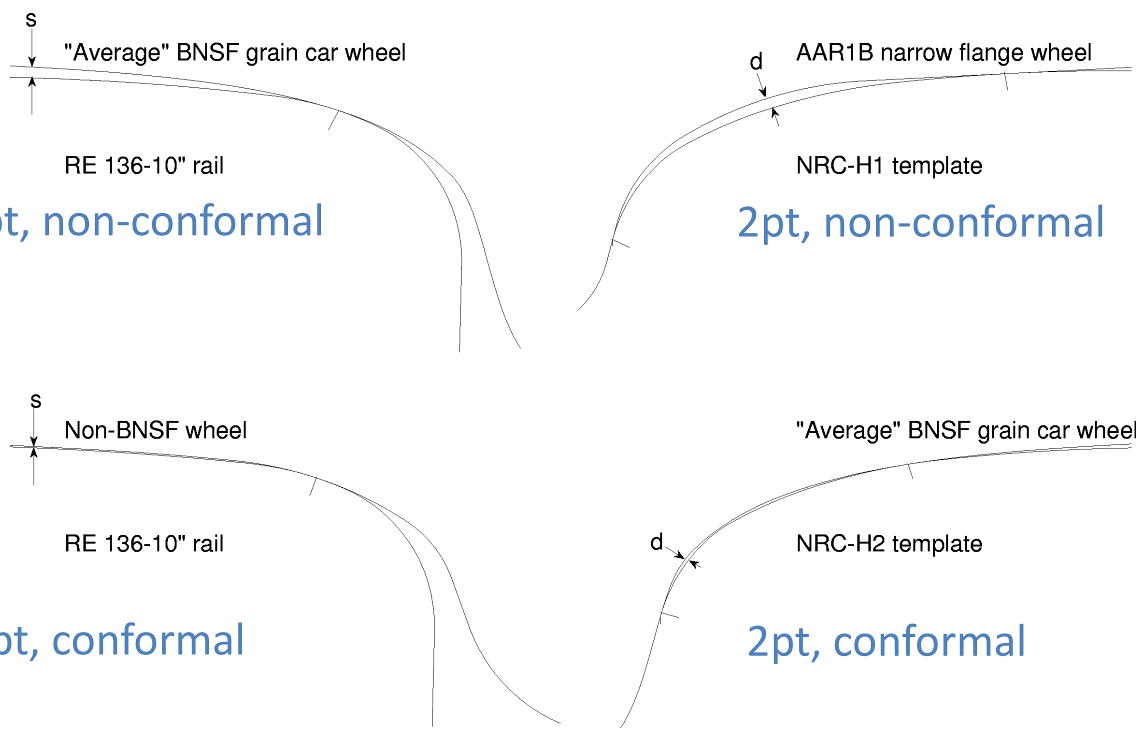


Worse steering

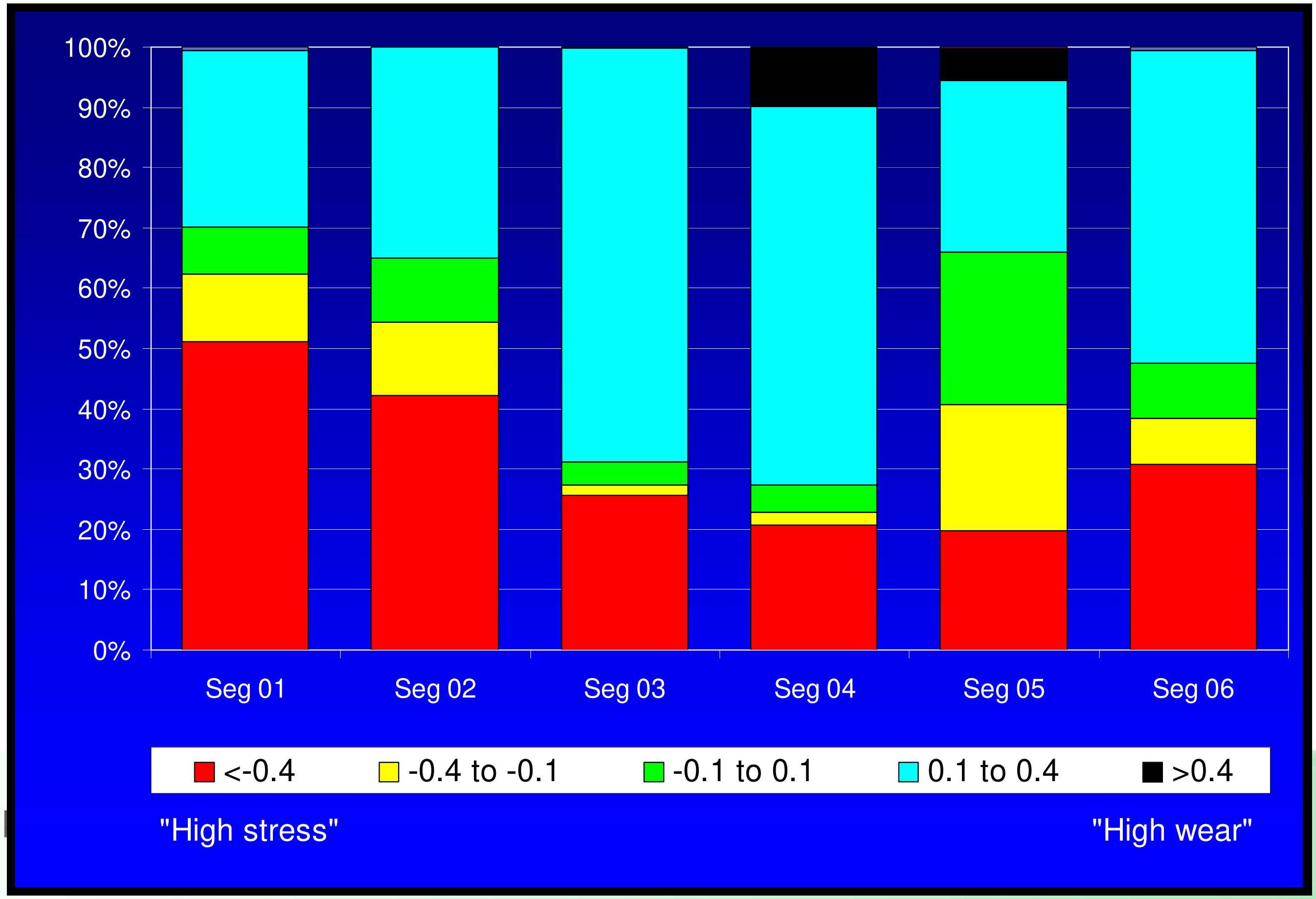


Better steering in curves

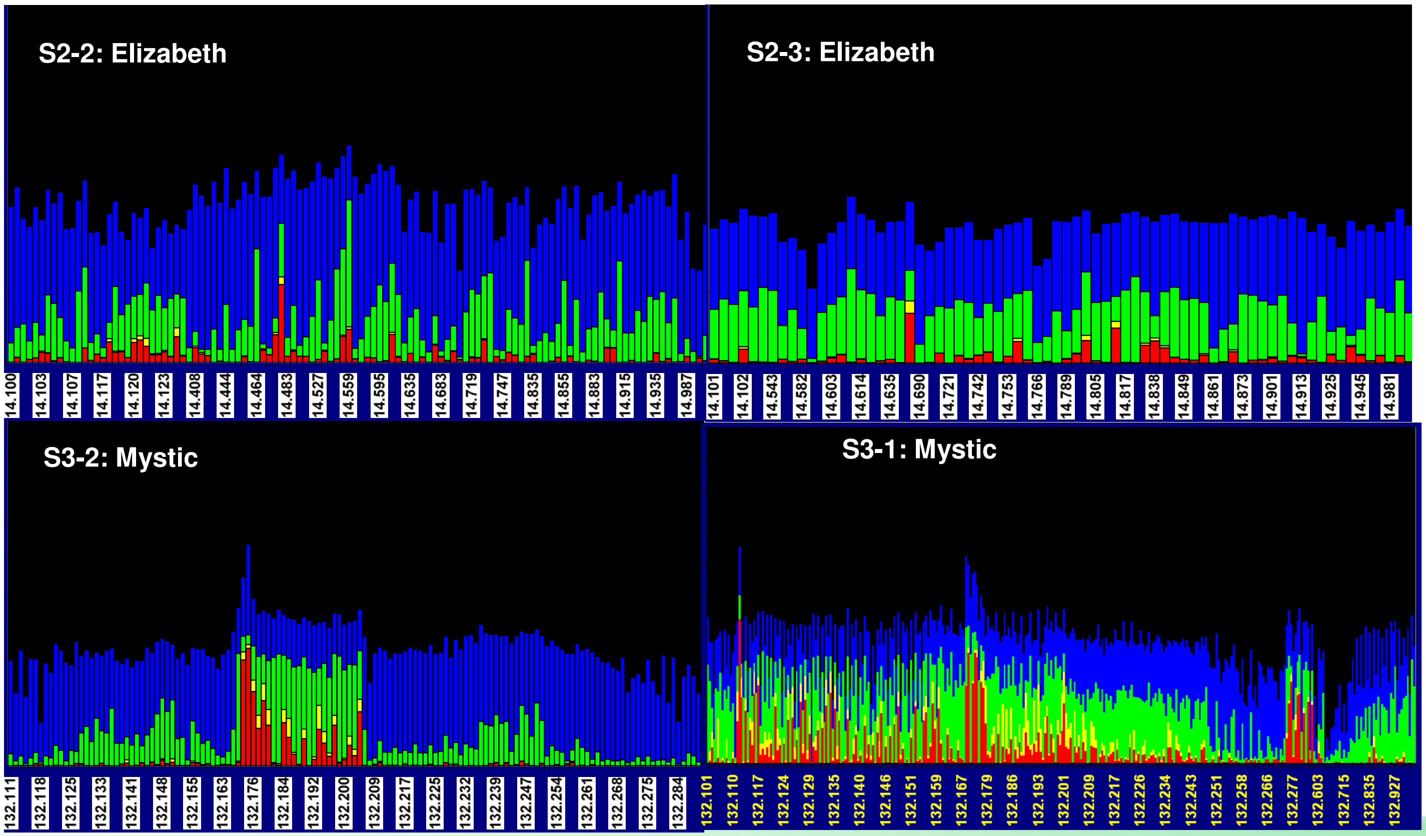
Wheel and rail profiles - conformality between the wheel and high rail



Conformality Analysis ™



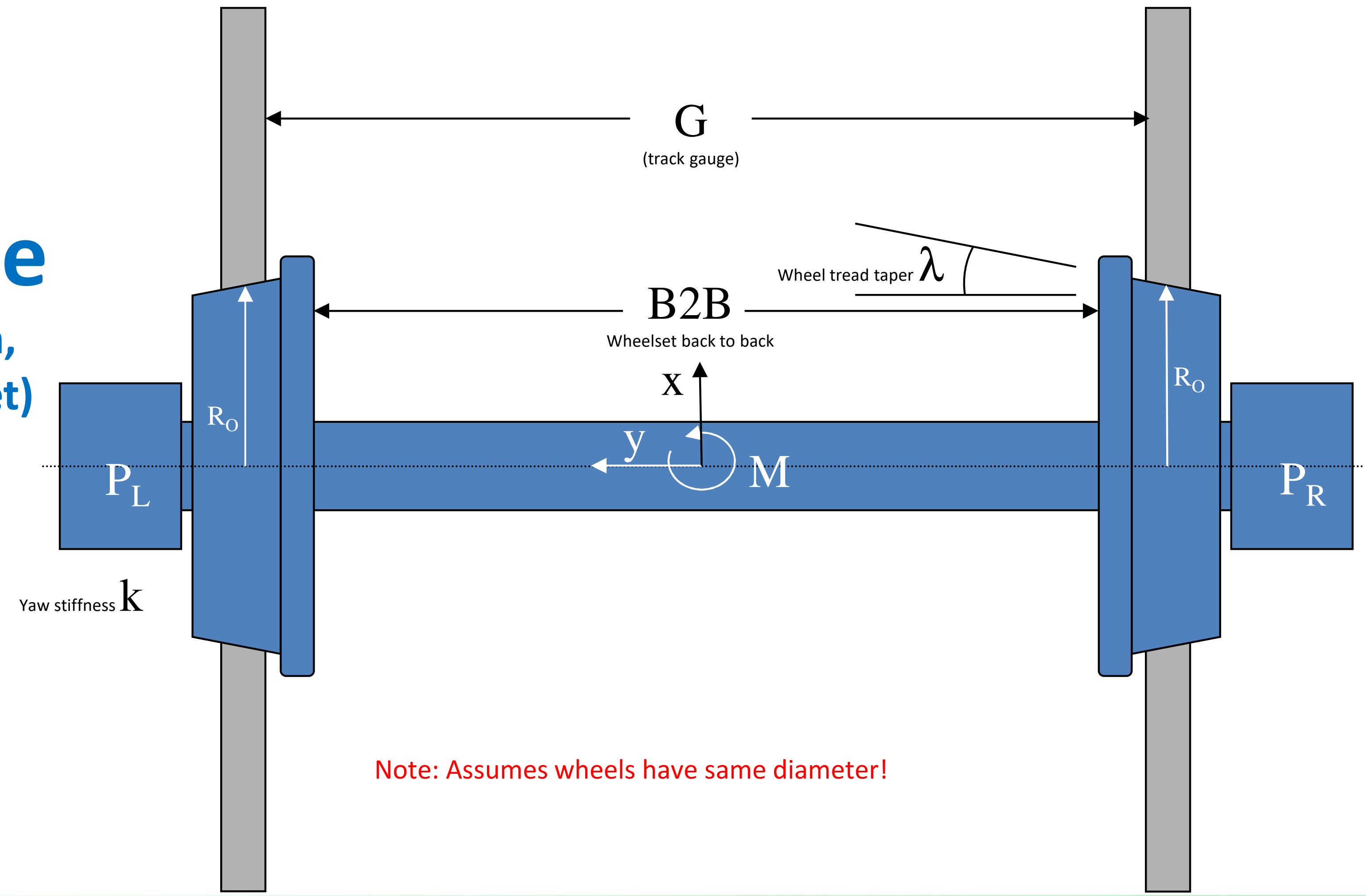
Conformality Analysis summary - sharp curves



EQUIVALENT CONICITY

Rolling Radius Difference

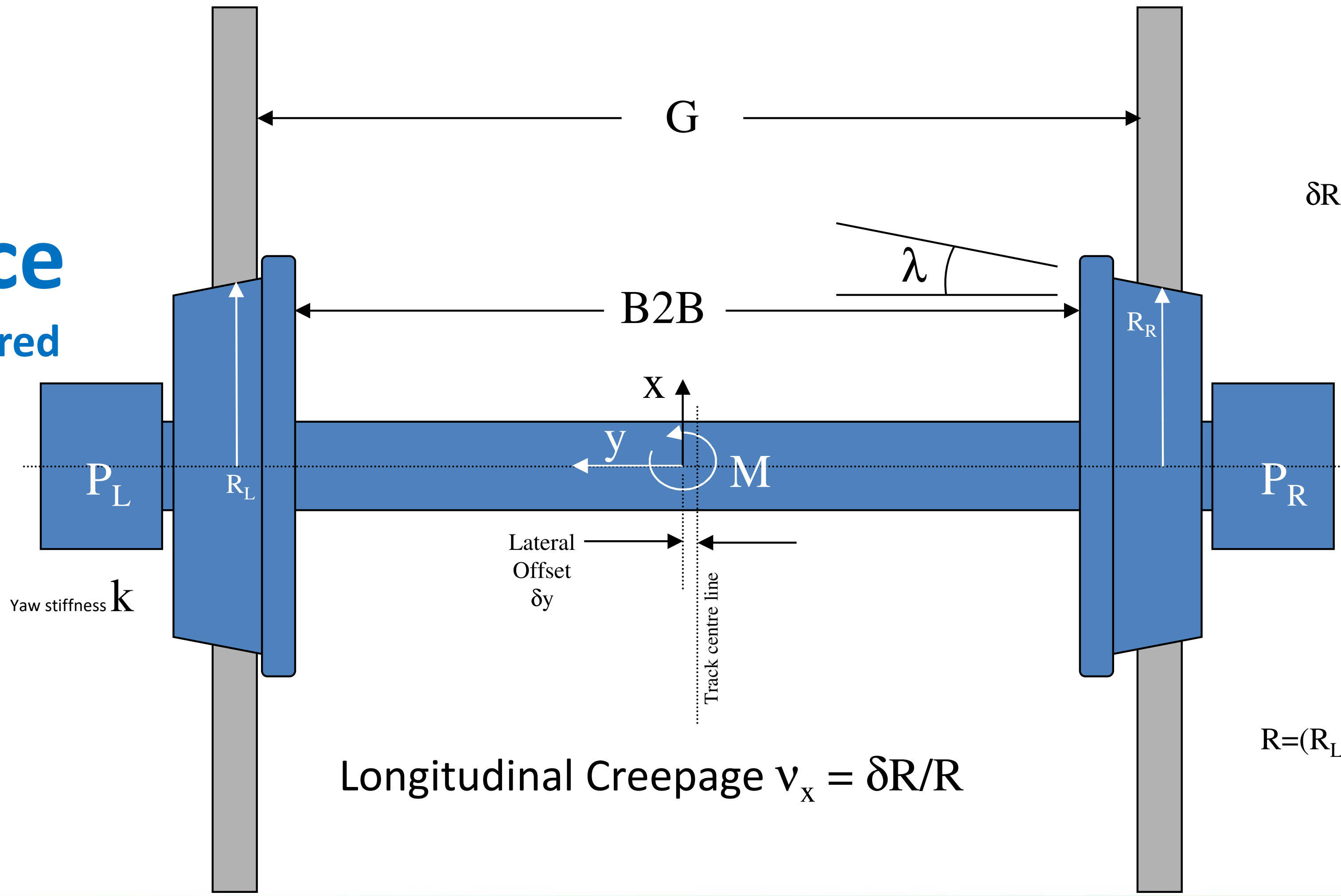
(zero for unworn, centered wheelset)



Note: Assumes wheels have same diameter!

Rolling Radius Difference

(for straight tapered wheel)

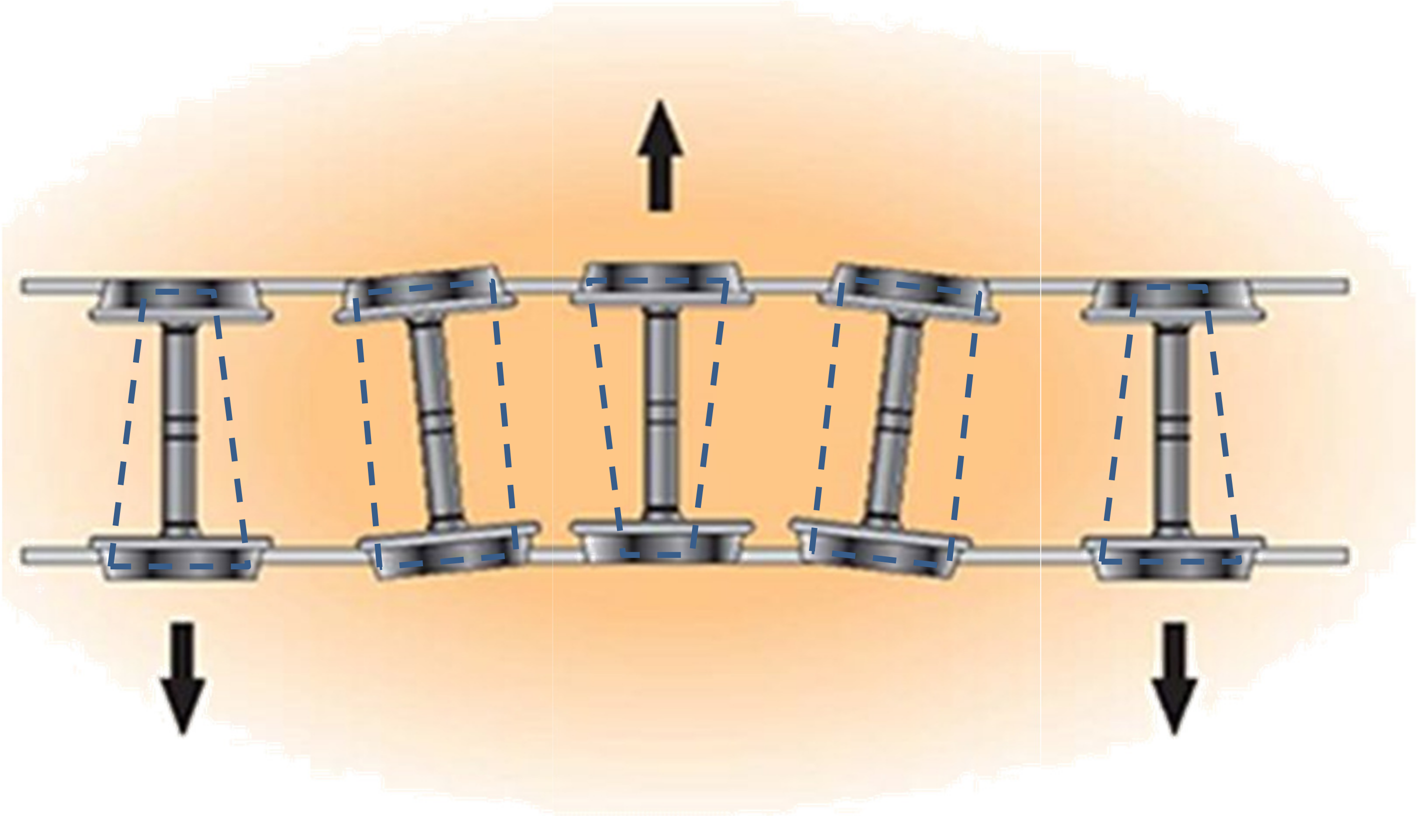


$\delta R = R_L - R_R$
$= \lambda \times \delta y$

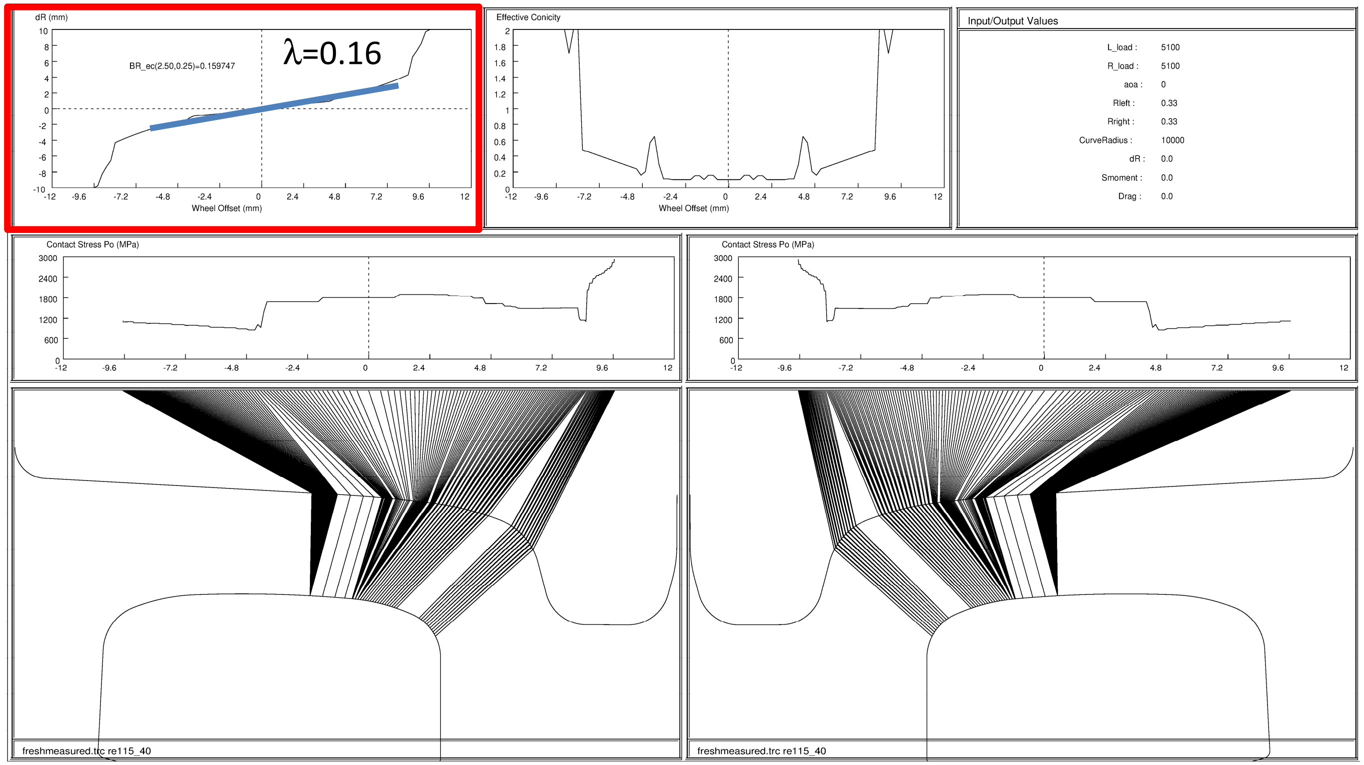
$R = (R_L + R_R)/2$

Longitudinal Creepage $\nu_x = \delta R / R$

The Free Wheelset - Hunting



Rolling Radius Difference



Why do we care?

Effects

- Hunting
- Lateral Forces
- Wheel/rail wear
- Wheel/rail RCF
- Corrugation
- Noise
- Vibration

Consequences

- Comfort, lading damage, safety
- Safety (Wheel climb, rail rollover)
- Economics, V/T availability
- Safety, inspection, maintenance
- V/T damage, maintenance
- Comfort, health
- V/T damage

Conclusions

- **Hertzian** contacts
  - Linear elasticity - line, point and elliptical contacts
    - These calculations are “reasonable”
    - Lesson: don’t rely too much on absolute numbers
- **Pummelling** – need to consider whole range of profiles/conditions borne by rail/wheel
- Roughness generally not a contributing factor re contact stress
- Wheel and rail (transverse) **profiles** control contact stress

Conclusions – cont’d

- Friction raises the stress levels (and damage) considerably
- **Stick and slip** regions in the contact patch
- The wheel most always slips on the rail
- **Negative Friction** is a root cause of much noise, vibration, corrugation
- **Shakedown**, **conformality** and effective **conicity** - useful methods to assess compatibility
- It is worth investing in contact mechanics to “get things right”

THANK YOU

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National Research Council, Canada